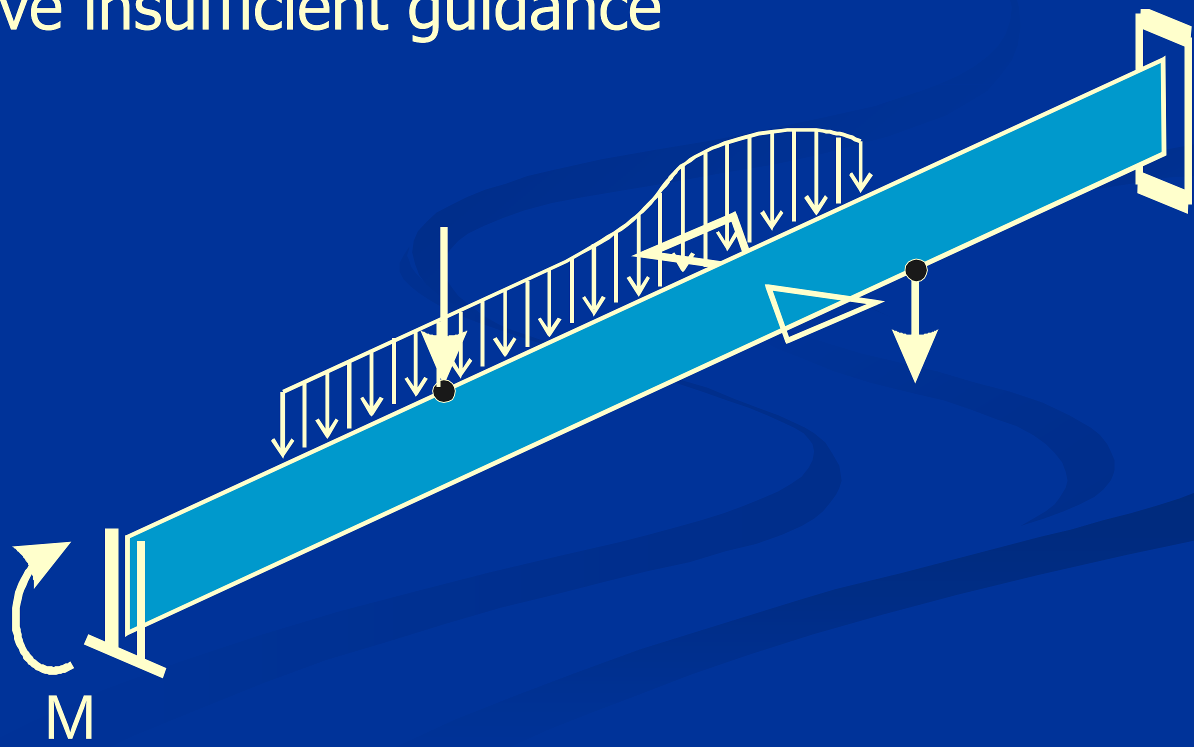
On the calculation of the critical moment to lateral-torsional buckling of beams: comparison of various methods

S. Ádány, A.L. Joó, D. Visy
Budapest University of Technology and Economics
Budapest, Hungary

Motivation

- Design for lateral-torsional (LT) buckling requires Mcr (e.g. EC3)
- Question: how to calculate critical moment (Mcr) ??
- Design codes give insufficient guidance



Methods for Mcr calculation

- Formulae
  - analytical
  - ENV version Eurocode 3, Annex F
  - AUS/NZ 4600

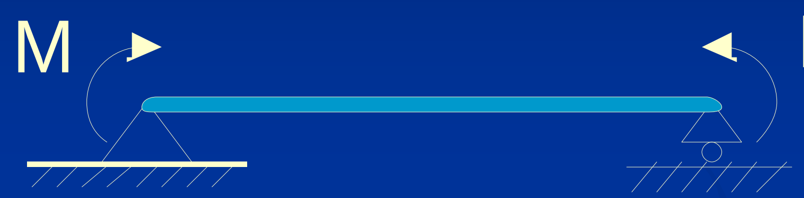
- Rational analysis
  - GBT – Generalized Beam Theory
  - FSM (cFSM) – (constrained) Finite Strip Method
  - FEM – Finite Element Method

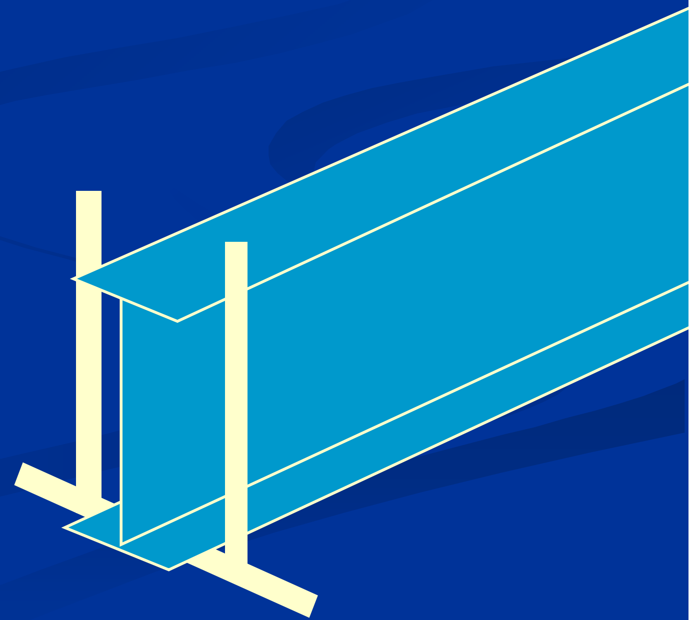
Outline

- Numerical studies: comparison of various methods
  - Study #1
  - Study #2
  - Study #3
- Conclusions

Study #1: subject, methods

- Double-symmetrical I-section (IPE400)
- Uniform moment
- Single-span beam
- „Fork” supports
- Considered methods:
  - Analytical formulae (=ENV=AUS/NZ)
  - GBT
  - cFSM
  - FEM



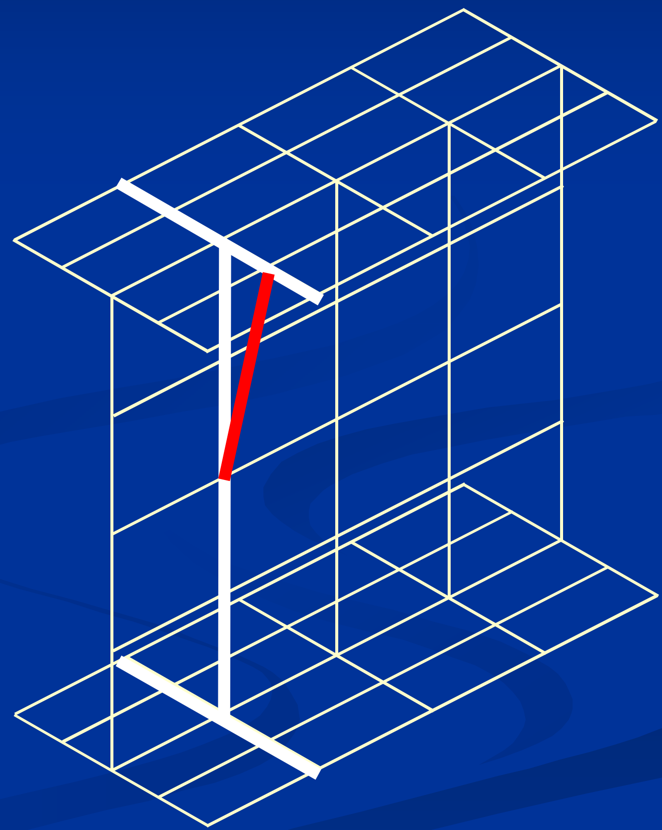


Study #1: FEM model

- Shell finite elements
- Ansys
- 3 types of shell elements:
  - SHELL63: 4-node, proposed for thin plates/shells, elastic analysis
  - SHELL181: 4-node, Mindlin-Reissner plate theory, proposed for moderately thick plates/shells
  - SHELL281: similar to SHELL181, but with 8 nodes
- Cross-section constraining by „diaphragms”
- Various discretizations – an „optimal” is used

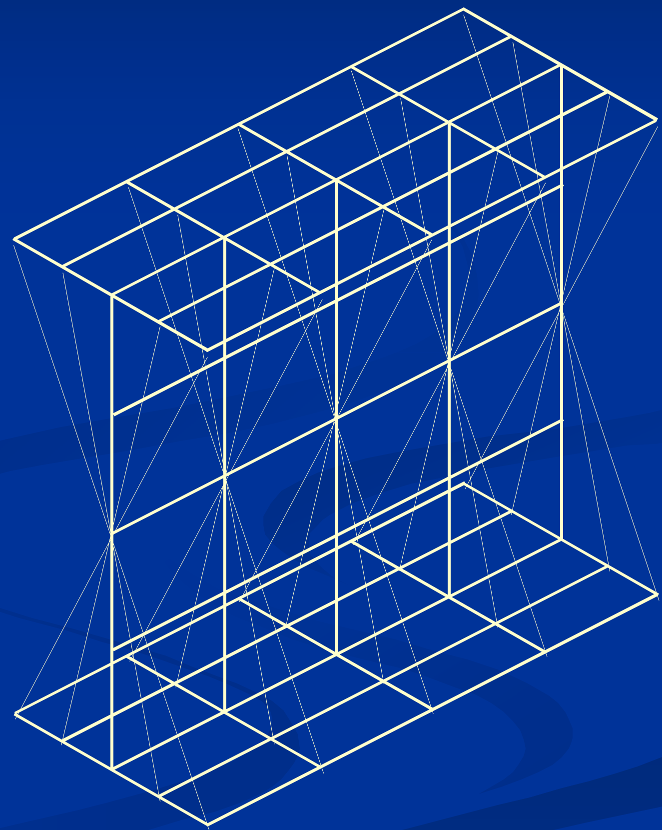
Study #1: Cross-section constraining

- Aim: to avoid cross-section distortion



Study #1: Cross-section constraining

- Aim: to avoid cross-section distortion
- Constraint equations
  - in Ansys: possible (CERIG command)
  - in simpler FEM software: not possible
  - decreased DOF number
- „Rigid” (truss) bars
  - possible in any FEM software
  - increased DOF number
  - simple way to control the position of direct transverse forces
- Not identical to a classical beam model !!



Study #1: Mcr values – comparison

- Normal steel material

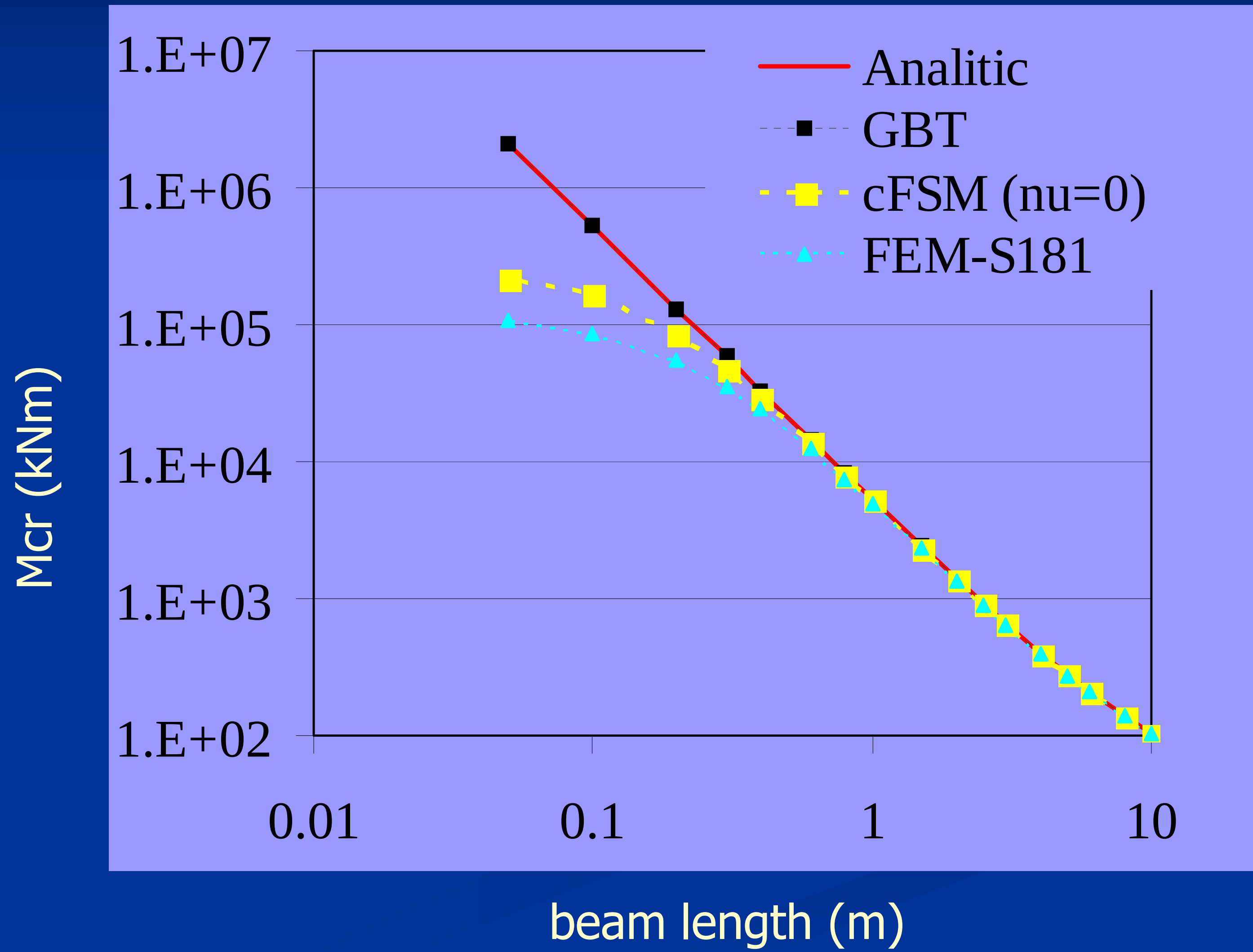
length (m)	FEM S63	FEM S181	FEM S281	cFSM	GBT	Analytic
10	105.05	103.23	103.53	111.62	105.25	105.20
3	648.71	641.87	649.20	715.33	659.17	658.85
1	4790.0	4918.4	4972.2	5736.6	5343.2	5340.6
0.4	20405	23823	--	31896	32984	32968

Study #1: Mcr values – comparison

- Normal steel material, but $\nu = 0$

length (m)	FEM S63	FEM S181	FEM S281	cFSM	GBT	Analytic
10	105.25	103.40	103.70	105.18	105.24	105.20
3	652.87	645.30	652.76	657.27	659.07	658.85
1	4867.0	4985.6	5041.0	5227.2	5342.4	5340.6
0.4	20173	24160	--	29031	32979	32968

Study #1: Comparison, cont'd

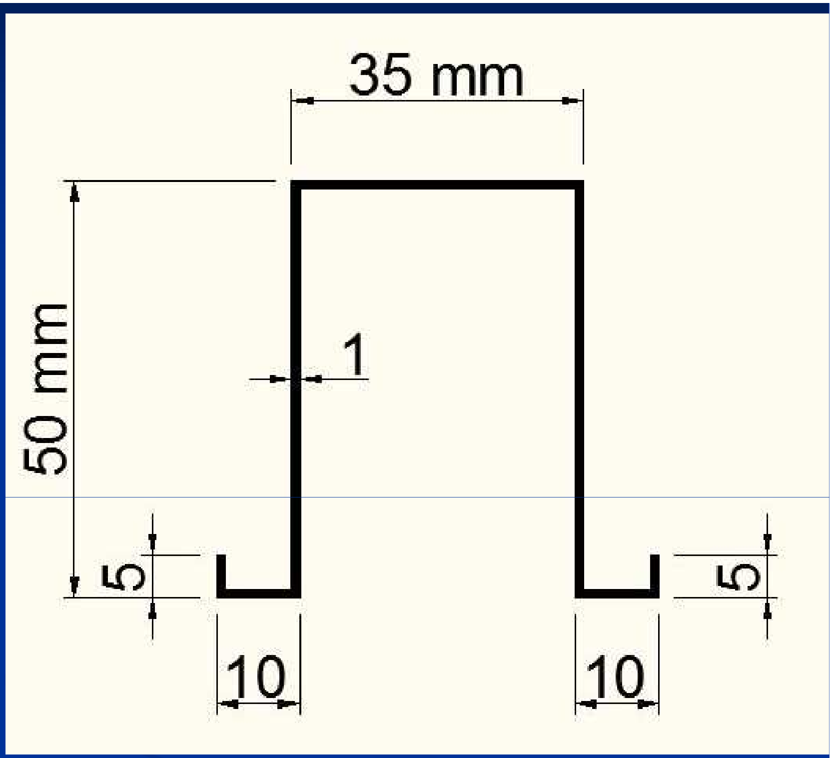


Study #1: Conclusions

- „Exact” value of Mcr cannot be defined even for the simplest case
- Very short beams: Mcr values are very much dependent on the method
  - FEM and cFSM are similar
  - GBT and analytical solutions are similar
- In case of FSM and FEM: constrained cross-sections + $\nu=0$
- In case of FEM: element type has min 2-3% effect

Study #2: subject, methods

- IPE400 and Hat section
- Linear moment diagram
- Single-span beam, various supports
- Considered methods:
  - FEM
  - GBT
  - ENV
  - AUS/NZ



Study #2: FEM model

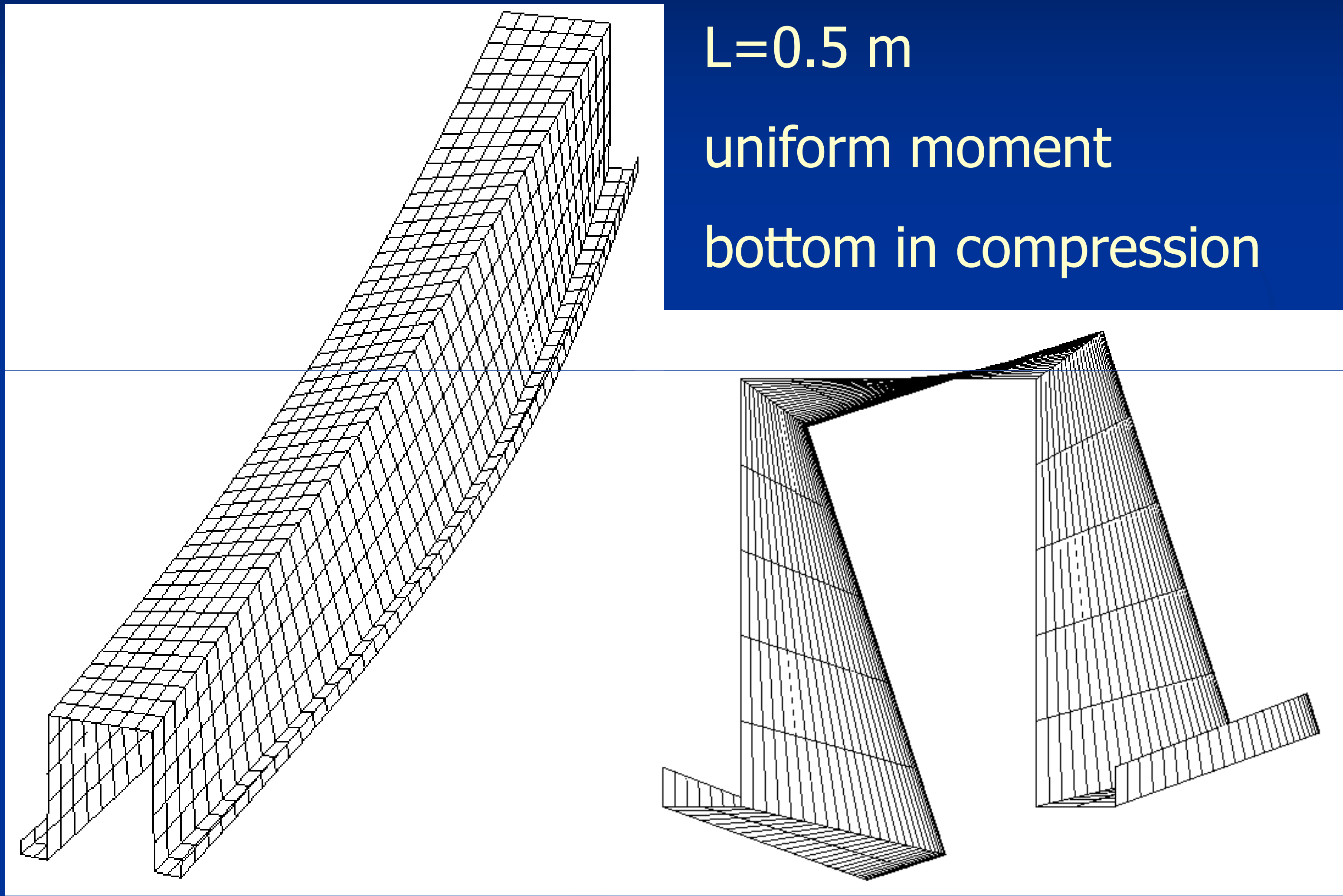
- Same as in Study #1
- FE type: SHELL181
- Cross-section constraining: constraint equations
- Medium dense FE mesh
- Material: normal steel, but $\nu=0$

Study #2: Some results

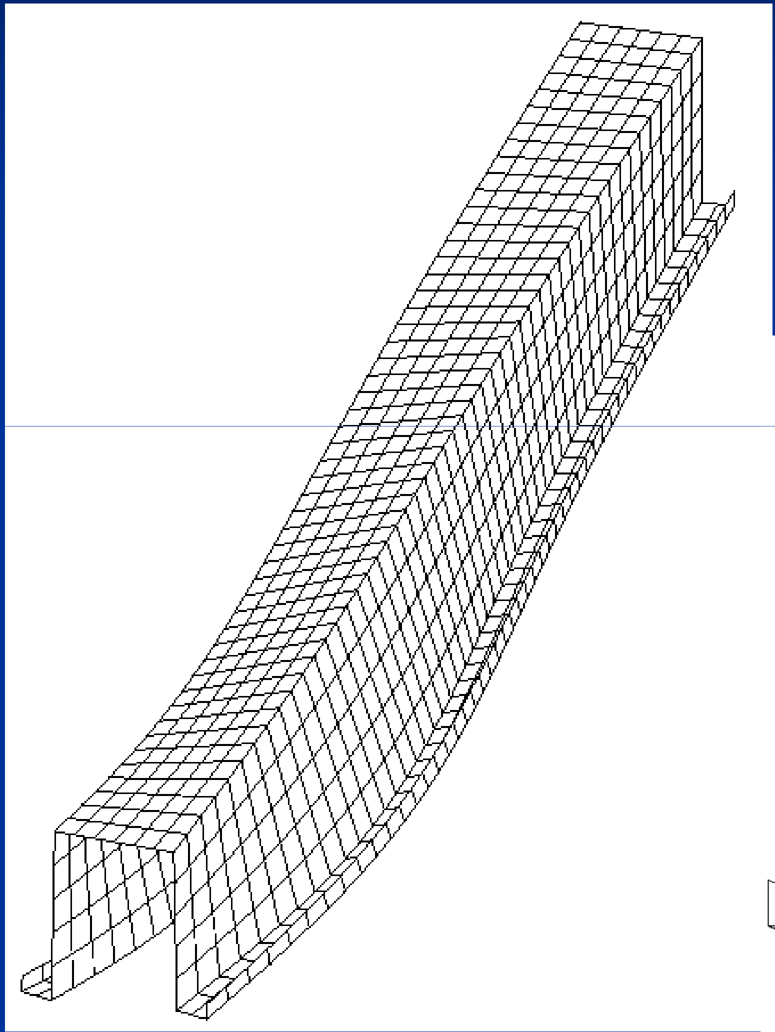
L=0.5 m

uniform moment

bottom in compression



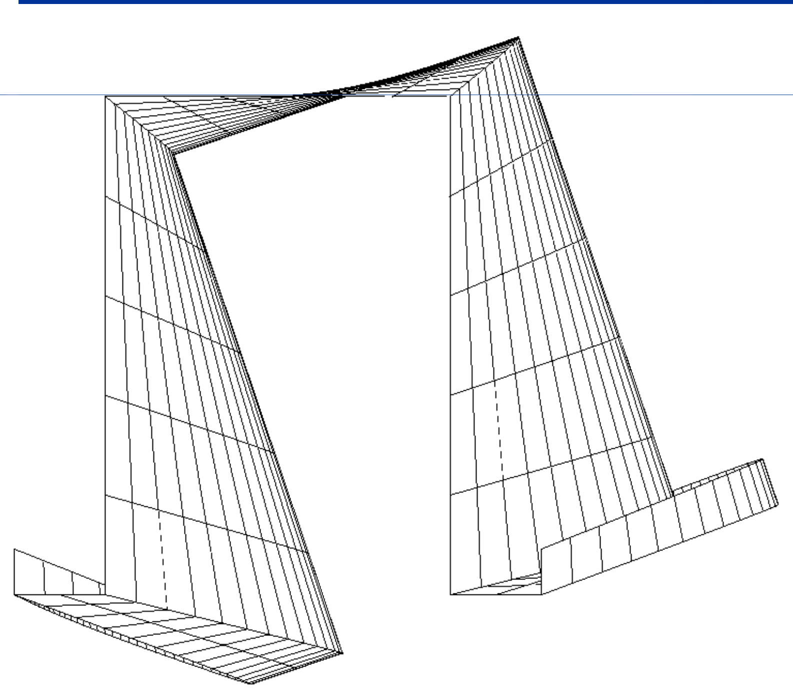
Study #2: Some results



L=0.5 m

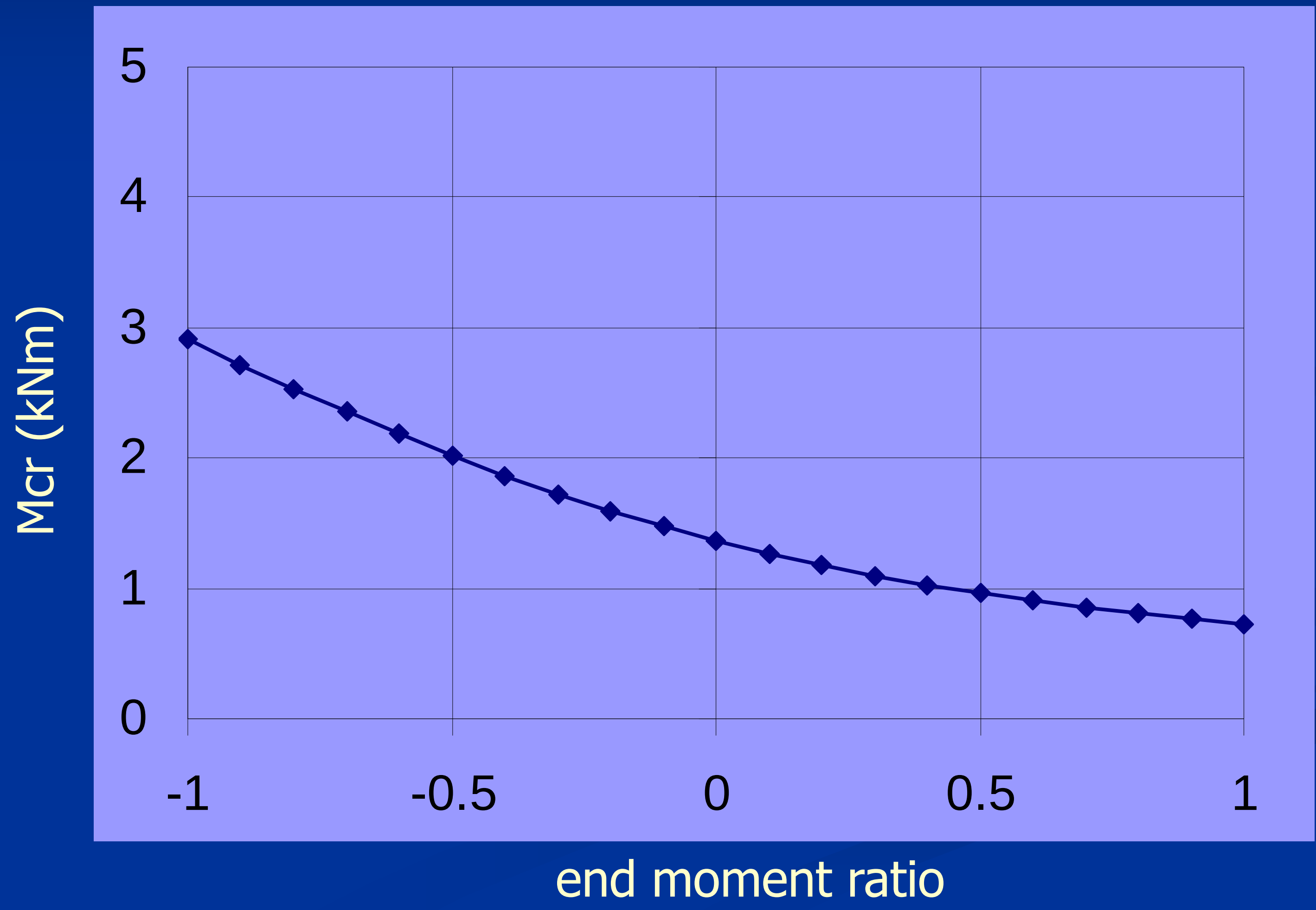
end moment ratio = -0.5

top is more compressed



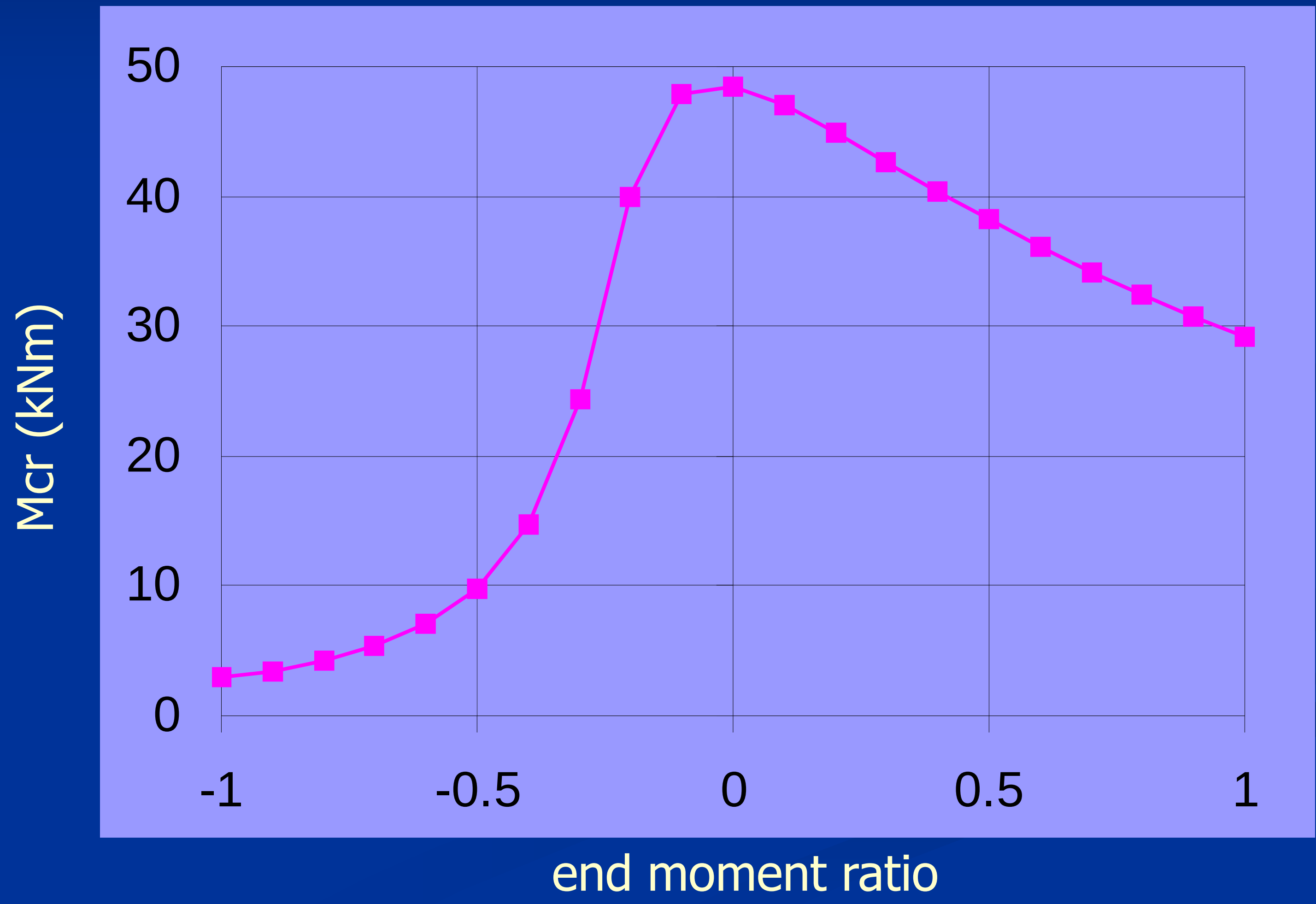
Study #2: Some results

- Hat section, bottom in compression



Study #2: Some results, cont'd

- Hat section, top in compression



Study #2: Comparison of various methods

- Hat section, fork supports, downward loading

mom. ratio	FEM (kNm)	GBT (%)	EC (%)	AUS (%)
1	0.1203	4.1	4.0	4.0
0.5	0.1586	4.1	5.2	-1.4
0	0.2204	4.2	13.2	-5.3
-0.5	0.3120	4.4	55.3	0.3
-1	0.4256	4.7	371	47.0

Study #2: Comparison, cont'd

- Hat section, fork supports, upward loading

mom. ratio	FEM (kNm)	GBT (%)	EC (%)	AUS (%)
1	4.0315	5.4	5.4	5.4
0.5	5.2966	5.3	5.3	0.3
0	6.8881	5.4	9.2	2.8
-0.5	1.3653	5.2	487	678
-1	0.4256	4.7	371	47.0

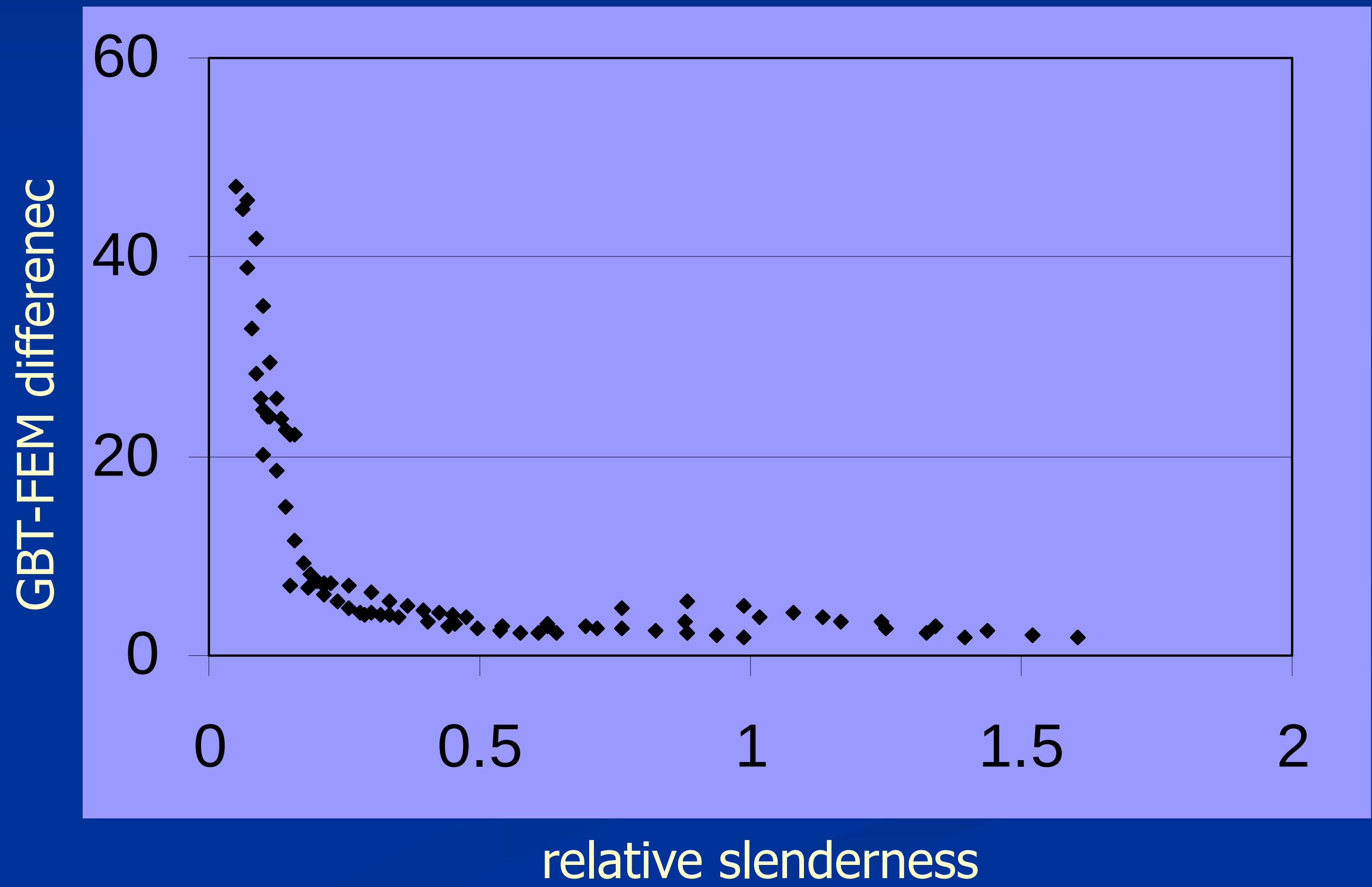
Study #2: Comparison, cont'd

- Hat section, partially clamped, upward loading

mom. ratio	FEM (kNm)	GBT (%)	EC (%)	AUS (%)
1	14.99	12.5	28.0	12.5
0.5	19.65	12.5	190	7.3
0	25.33	13.2	202	11.0
-0.5	5.545	6.5	1336	660
-1	1.520	4.2	421	23.3

Study #2: GBT-FEM difference

- various end moment ratios, lengths



Study #2: Conclusions

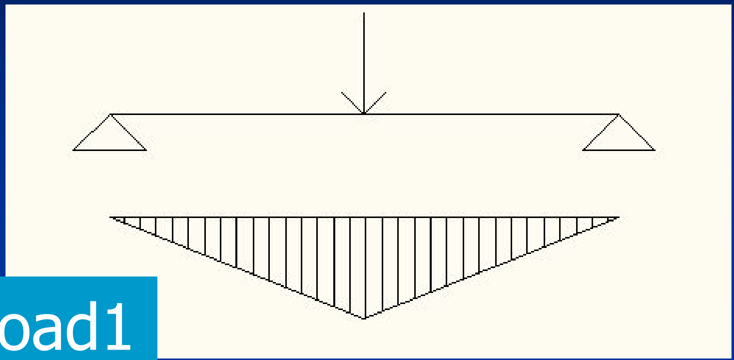
- GBT and FEM
  - good coincidence for practical cases
  - different tendencies for very small slenderness

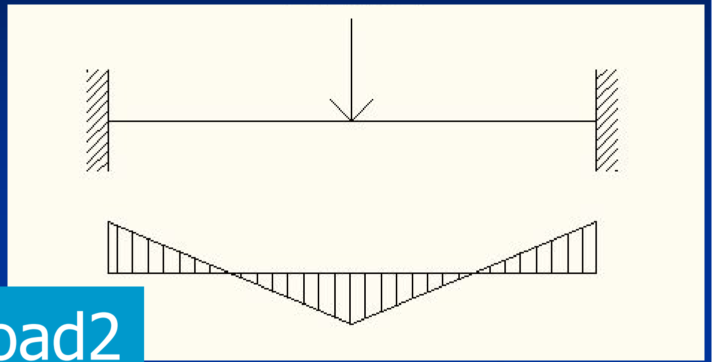
- ENV and AUS/NZ formulae
  - frequently lead to very bad Mcr estimations
  - reasonable results only for limited cases
    (e.g. double-symmetrical cross-sections, fork supports,
    end moment ratio is positive)

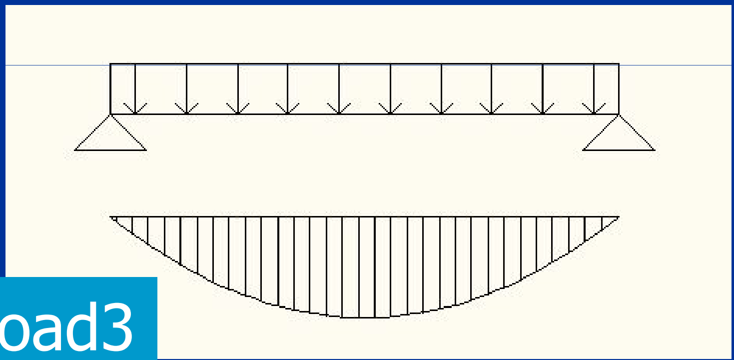
Study #3: subject, methods

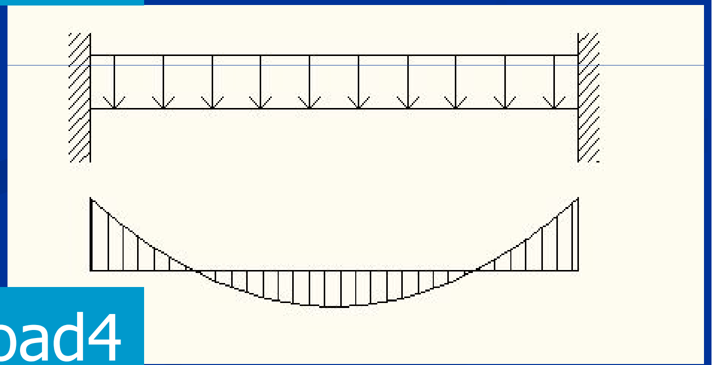
- IPE400 and Hat section
- single-span beam,
  various supports
- with transverse loading
- Considered methods:
  - FEM (same as in Study #2)
  - GBT
  - ENV (but no AUS/NZ)

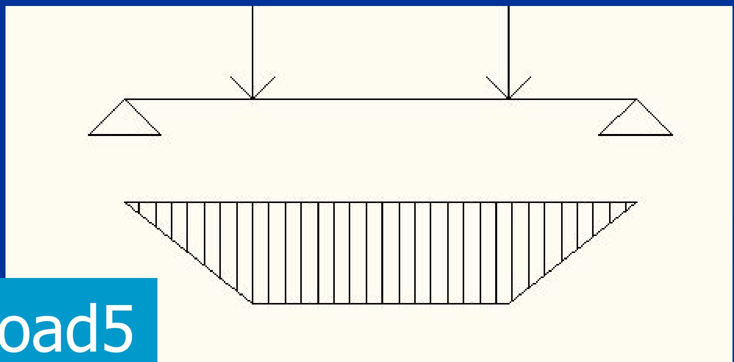
Study #3: loading


Load1

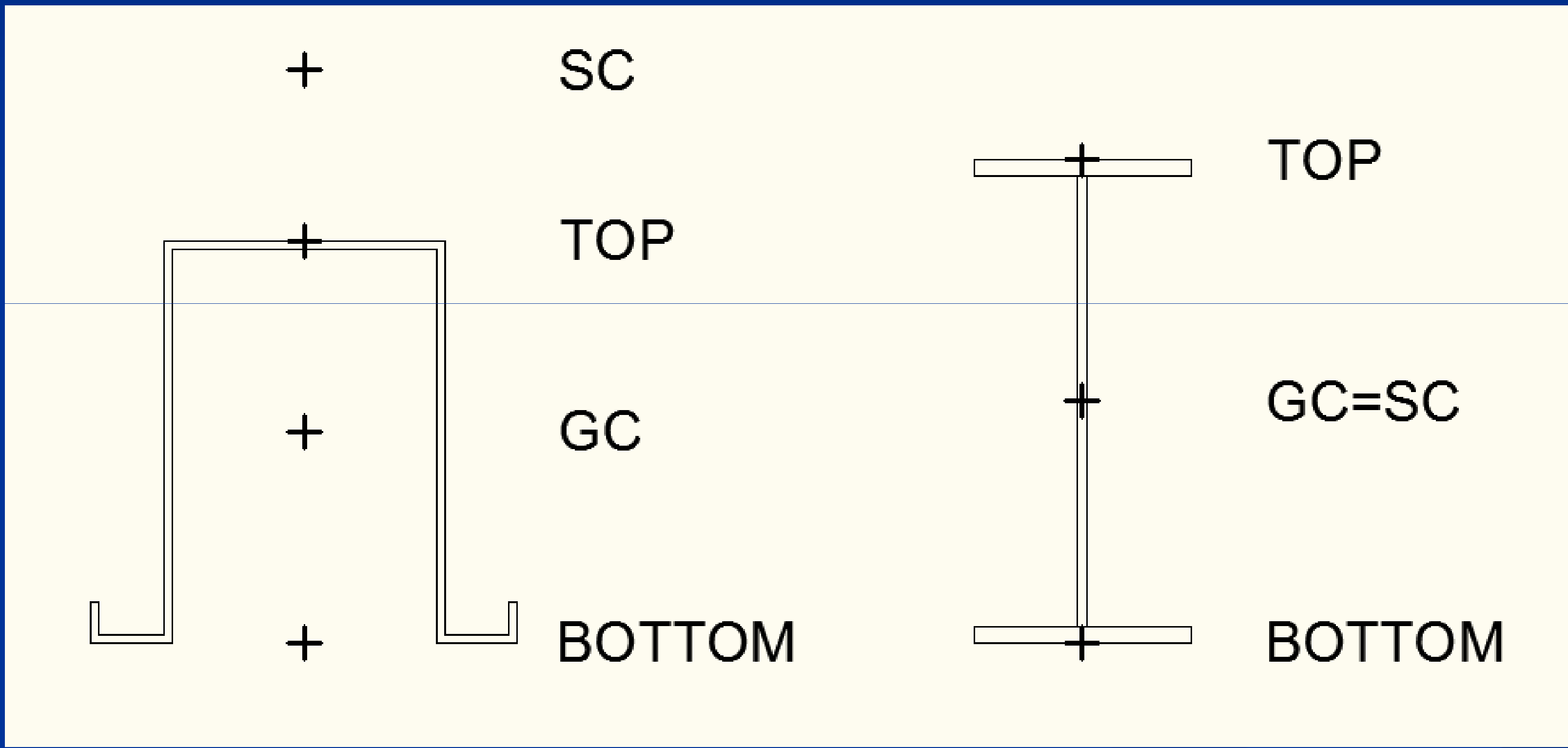

Load2


Load3


Load4

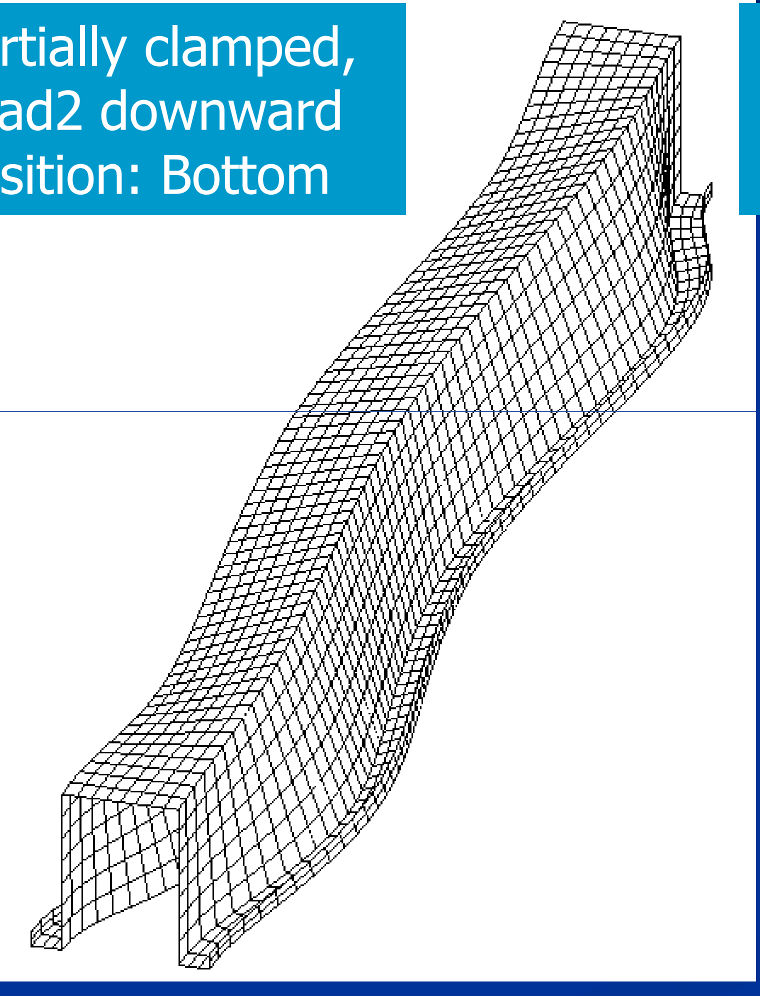

Load5

Study #3: load application points

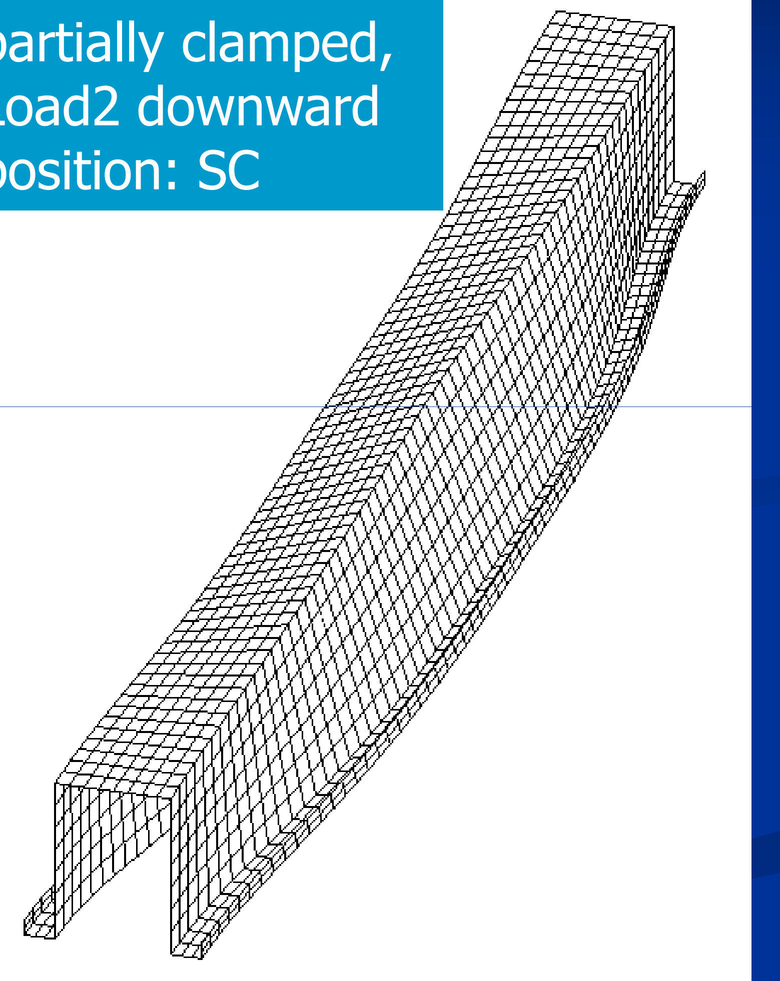


Study #3: Some results

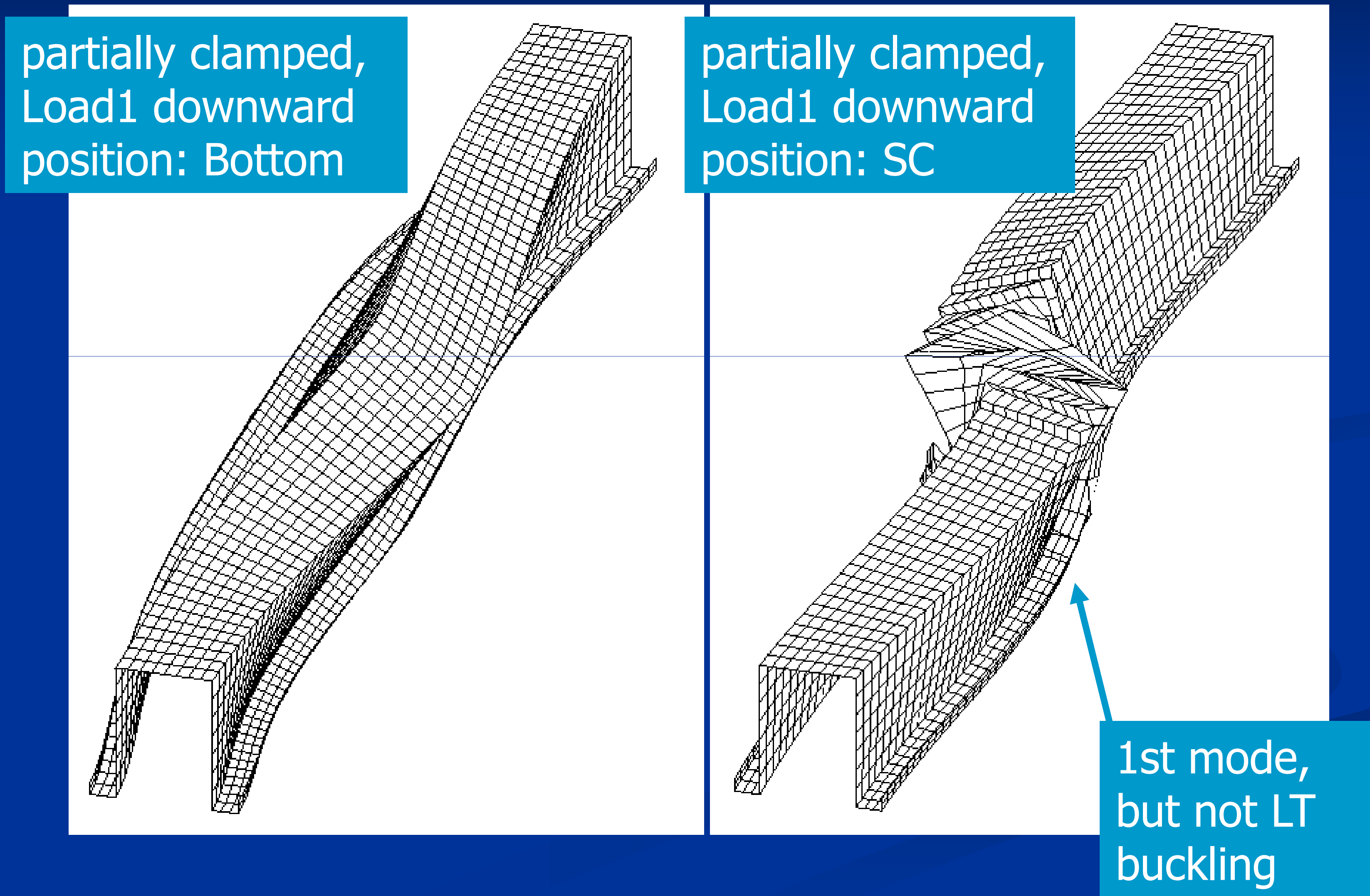
partially clamped,
Load2 downward
position: Bottom



partially clamped,
Load2 downward
position: SC



Study #3: Some results, cont'd



Study #3: Comparison of various methods

- IPE, L=10 m

load type	load appl. point	fork FEM (kNm)	fork GBT (%)	fork EC (%)	part. clamped FEM (kNm)	part. clamped GBT (%)	part. clamped EC (%)	part. clamped FEM (kNm)	part. clamped GBT (%)	part. clamped EC (%)
Load1	Top	105		3.3	157		-2.3	207		3.2
	GC	141	1.8	2.1	198	1.9	-3.9	291	1.9	2.3
	Bottom	188		1.5	248		-5.1	408		1.7
Load2	Top	89		0.9	135		-4.1	153		1.3
	GC	165	2.0	-7.2	218	2.0	-11.6	261	2.1	-8.9
	Bottom	305		-12.9	351		-17.6	445		-16.1
Load3	Top	93		1.6	144		-8.5	209		2.1
	GC	118	1.3	1.3	168	1.3	-6.4	266	1.5	1.6
	Bottom	149		1.2	197		-4.2	336		2.0
Load4	Top	127		1.3	231		-13.2	256		-5.3
	GC	273	0.5	-0.9	387	0.4	-7.5	479	1.0	-17.2
	Bottom	572		-1.0	639		-0.1	880		-26.4
Load5	Top	86		2.3	139		-11.4	216		-5.1
	GC	107		2.4	156		-6.7	258		9.0
	Bottom	132		3.6	174		-0.9	298		29.4

Study #3: Comparison of various methods

- Hat section, L=1.5 m

load type	load dir.	load appl. point	fork FEM (kNm)	fork GBT (%)	fork EC (%)	part. clamped FEM (kNm)	part. clamped GBT (%)	part. clamped EC (%)	part. clamped FEM (kNm)	part. clamped GBT (%)	part. clamped EC (%)
Load1	+	SC	2.39		312	3.89		158	7.65		605
	+	Top	3.42		227	4.92		131	10.3		454
	+	GC	4.58	6.5	178	6.08	8.1	111	13.4	15.2	354
	+	Bottom	5.81		147	7.33		98.0	16.8		287
	-	SC	0.34		-70.1	0.62		-52.9	0.66		-79.6
	-	Top	0.23		-62.1	0.49		-47.3	0.51		-75.2
	-	GC	0.17	-0.6	-54.7	0.39	0.9	-41.9	0.40	0.8	-70.6
	-	Bottom	0.13		-48.1	0.32		-37.0	0.33		-66.0
Load2	+	SC	0.26		6497	1.40		1133	1.47		4955
	+	Top	0.48		4245	2.43		757	2.55		3000
	+	GC	0.79	11.6	3043	3.44	10.2	620	3.53	10.2	2295
	+	Bottom	1.02		2755	4.10		611	4.14		2089
	-	SC	1.42		-94.6	1.39		-83.8	1.46		-94.9
	-	Top	0.58		-89.1	0.73		-74.5	0.75		-90.6
	-	GC	0.25	-7.0	-78.6	0.43	-1.1	-63.9	0.44	-1.1	-84.9
	-	Bottom	0.14		-67.5	0.29		-54.3	0.29		-78.9

Study #3: Conclusions

- good coincidence between GBT and FEM for practical cases
- limits w.r.t loading in GBTUL
- due to direct transverse forces first buckling mode is not always local even if cross-sections are constrained

Conclusions

- „Exact” value of Mcr ??
- GBT and FEM can be proposed for Mcr calculation
- FEM is more general, but its proper application is demanding
  - cross-sections are to be constrained (plus $\nu=0$)
  - excluding non-LT buckling is difficult for thin plates
- Applicability of ENV and AUS/NZ formulae is limited
- Formulae may lead to significant errors
- Very short beams: shell-type and beam-type numerical models have different tendencies
- More results for C sections

Thank You!