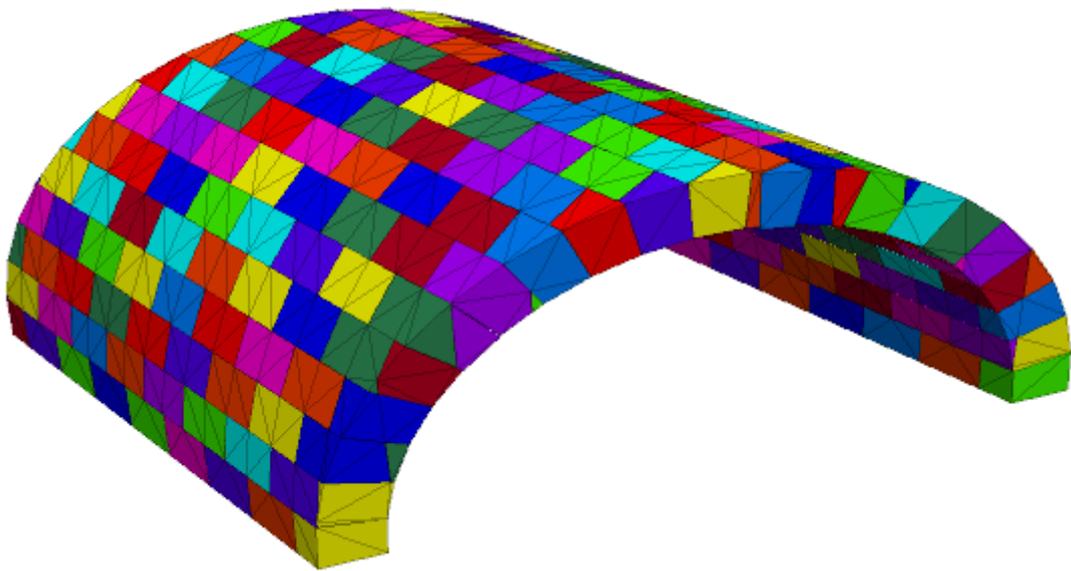


COMPARISON OF REGULAR AND SKEW ARCH BRIDGES



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ABSTRACT

The work reported in this homework summarizes the main differences between a regular and a skew masonry arch bridge. The results are obtained from a 3D computational model, using the distinct element software 3DEC. The variable investigated in this report was the angle of skew. In order to gain an understanding of the behaviour of the arches, no attempts were made to model the effects of fill, spandrel walls or any construction detail. For each model only the effect of self-weight was investigated. The results of skew arches are compared with similar regular arch in order to identify the influence of skew on the behaviour of the arches.

INTRODUCTION

An old and interesting structure which may still be seen in service today is the voussoir arch. This structure is composed of many individual sections, called voussoirs, which are simply fitted one upon another to form an arch.

A skew arch is a method of construction that enables masonry arch bridges to span obstacles at an angle. According to [2] bridges with a small amount of skew (less than 30°) can be constructed using bedding planes parallel to the abutments. However, bridges with large amount of skew present significant construction difficulties. Figure 1 presents three possible methods of construction of a segmental arch spanning a 45 degrees of skew.

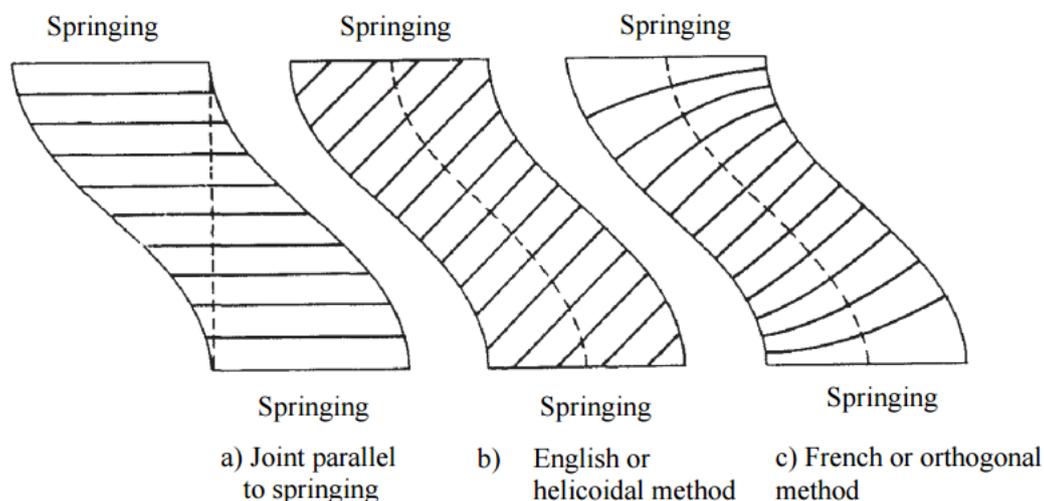


Figure 1: Intrados of an arch spanning at 45° skew [1]

According to [2] there are many thousands of stone masonry arch bridges in Europe, many of which have spans with a varying amount of skew. Most of these bridges are well over 100 years old and are supporting traffic loads many times above those originally envisaged.

There is an increasing demand for a better understanding of the life expectancy of such bridges in order to maintain, repair and strengthening strategies. Although a great deal of work has been carried out to assess the strength of square span masonry arch bridges using 2D

methods of analysis, comparatively little work has been undertaken to understand the three dimensional behaviour of skew arches [2].

In this homework Distinct Element Method was used to investigate these arches. With this method the discrete nature of the masonry arch is incorporated. The DEM was initially developed by Cundall to model blocky-rock systems and sliding along rock mass. The approach was later used to model masonry structures including arches, where failure occurs along mortar joints.

The aim of this homework is to receive a first impression on the differences in the mechanical behaviour of a straight and a regular arch.

SOLUTION STRATEGY

Unfortunately 3DEC doesn't have user friendly graphical interface. So usually the users have to design the geometry with other programs. Therefore I made the geometry with the help of MAPLE 15. All of the properties of the arch are programmed parametrically, so it is very easy to modify the model.

When I had the proper geometry I generate finite element mesh, then I set up boundary conditions. After it I applied the external load (in this homework only the self-weight was considered). In the end the program searched for the equilibrium state.

DISCRETE ELEMENT MODEL

I built more discrete element model in 3DEC 5.00.164, which were different in the sense of the angle of skew.

GEOMETRY

According to my homework description the radius of the intrados is 2m, the thickness of the arch is 40 cm. The width of the arch is 4m.

The following figures represent the basic parameters, the applied coordinate system and the numbering method of the nodes:

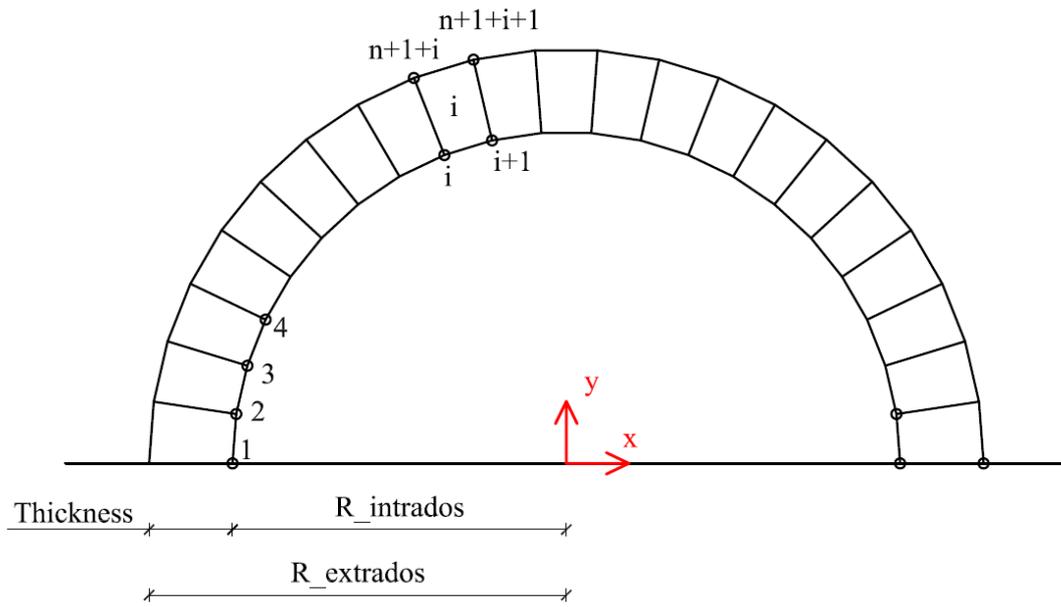


Figure 2: Front view of the arch

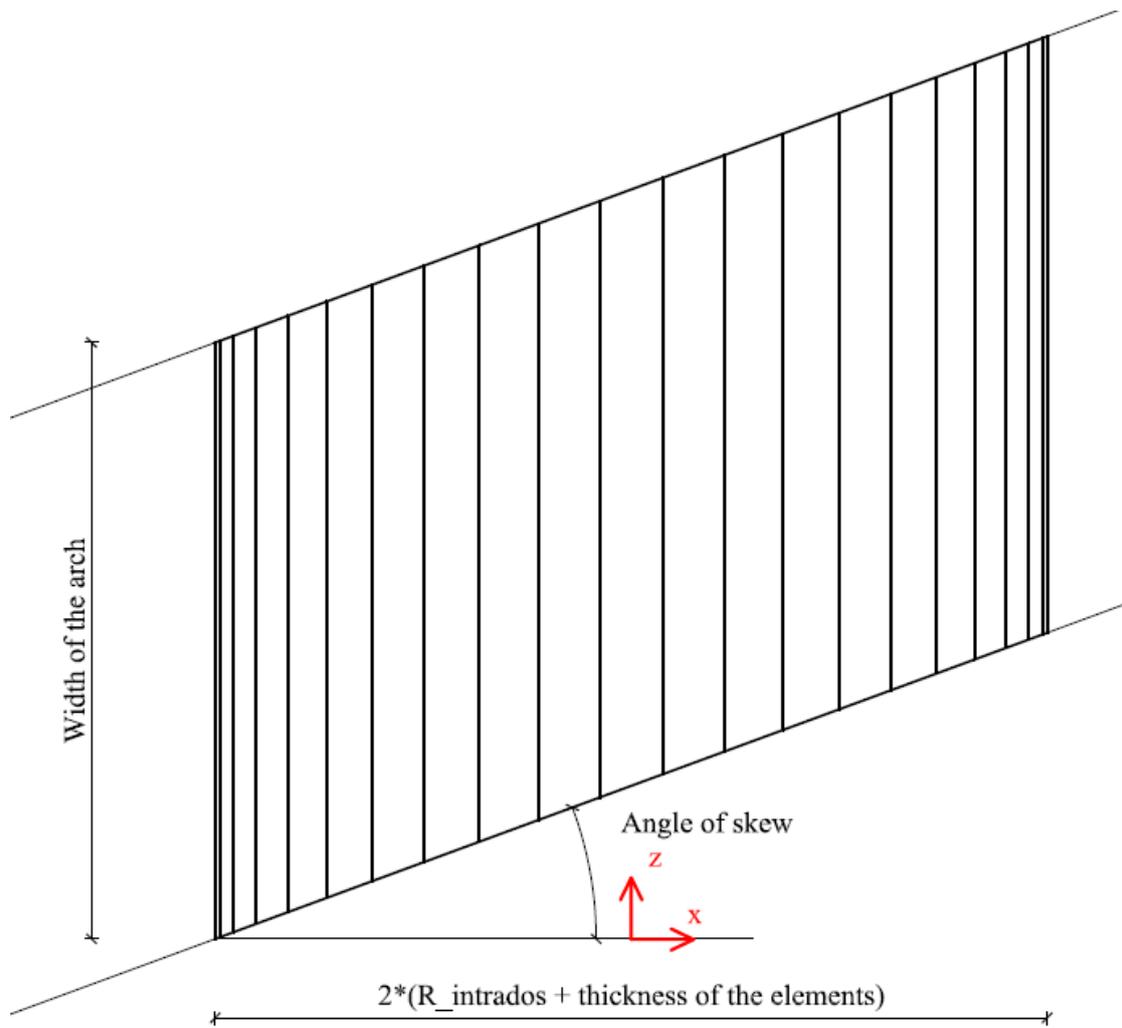


Figure 3: Top view of the arch

MATERIAL MODELS OF THE BLOCKS

I applied deformable elements in the models. According to the homework assignment the arches were made from sandstone.

Material	Density	Bulk moduli	Shear moduli	Young's moduli	Friction angle
Sandstone	2600 kg/m ³	26,8GPa	7GPa	18,2GPa	40 degree

Table 1: Applied material properties

CONTACT PROPERTIES

I used Coulomb frictional contacts in my models, so rigid, tensionless contact with high friction. The shear stiffness (jks) and the normal stiffness (jkn) are 10¹² N/m². According to the assignment I used 40° as the friction angle.

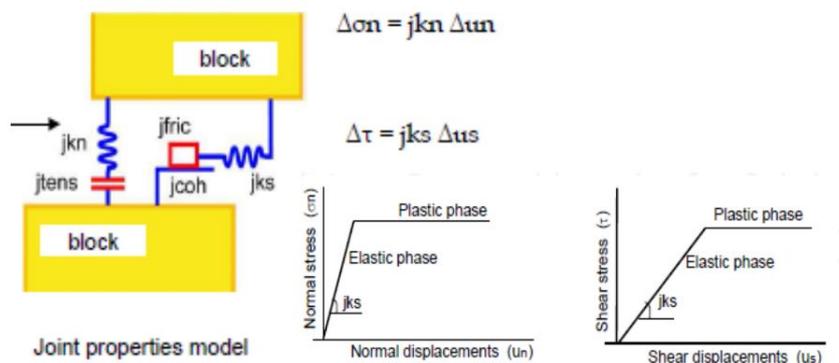


Figure 4: Contact properties in 3DEC [3]

BOUNDARY CONDITIONS AND FEM MESH

The arches were supported from below by a big block which was fixed. To simulate plane strain state I applied two vertical support blocks, which were fixed but have frictionless contacts. In the case of skew arches these vertical walls don't ensure plane strain state. In this case the user should apply the "boundary xvel" command.

In the model I used deformable elements. In 3DEC if we want to use deformable elements, then all the elements must be deformable.

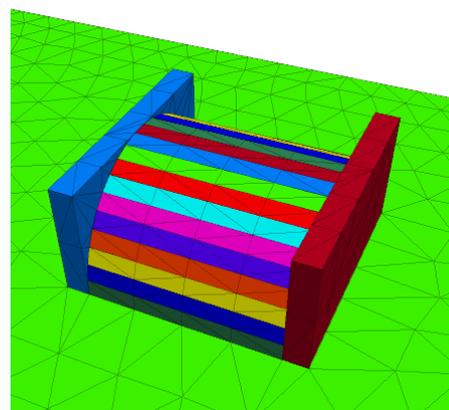


Figure 5: Boundary conditions + FEM mesh

LOADING

The applied load was the self-weight which can be apply by the gravity command.

RESULTS

REGULAR ARCH

After applying the gravity the program converges to the solution very fast. After 2000 cycles the unbalanced forces were under 10 N. It took only one minute computational time.

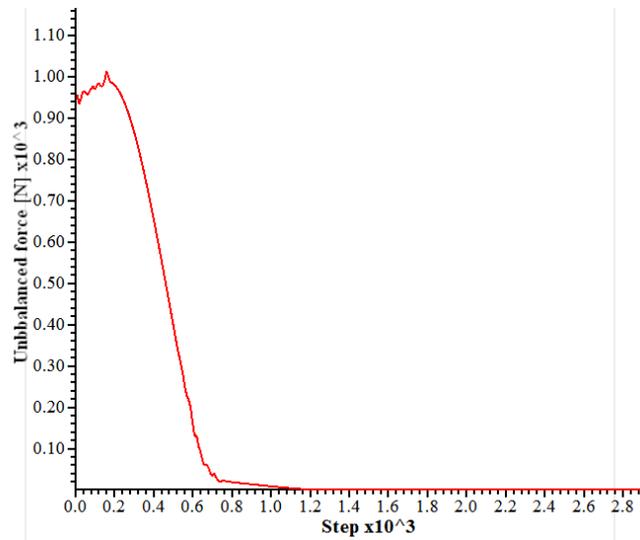


Figure 6: Unbalanced forces of regular arch

Because the structure and the applied load are symmetric, the displacements have to be also symmetric. The deflection of the midpoint was around 0,03 mm.

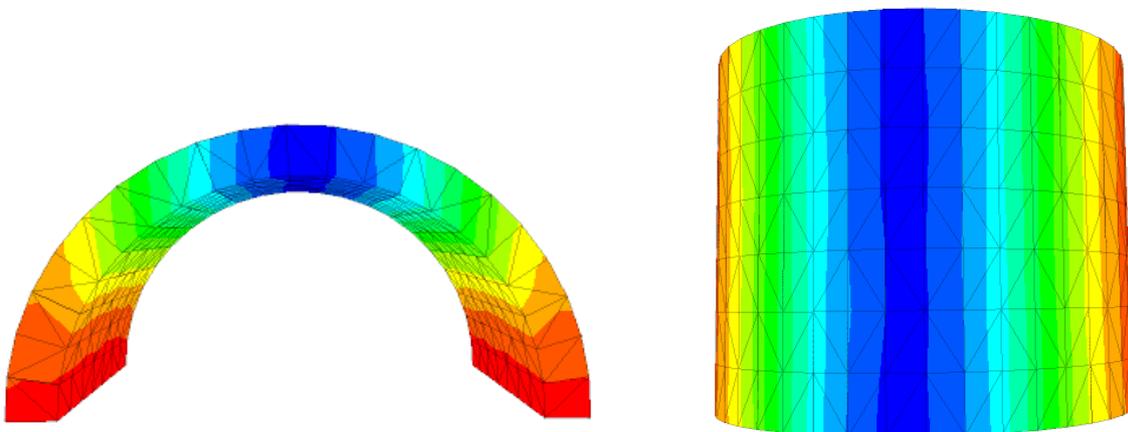


Figure 7: Vertical displacements of a regular arch – front and top view.

Several problems came out when I investigated the contact forces. The program shows us the normal and the shear forces acting on the contact if we use the “plot jointcontour nforce” command. But the plotted values are some kind of subcontact forces, they are not stresses. These values depend on the finite element mesh, if we use smaller mesh, then the nforce values are changed. The results are hardly interpretable.

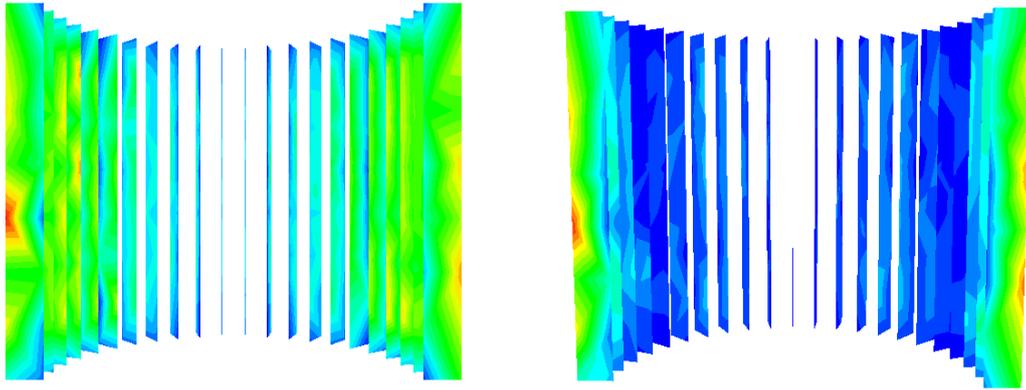


Figure 8: Contact forces – normal and shear forces (hardly treatable)

Finally I found a different plot which shows the normal and the shear stresses on the contact elements. In my opinion the theoretically proper distribution would be a uniform distributed and symmetric (around zy plane) contact force diagram. The blue color means 0 normal forces, so the faces of the blocks separated here. The red color means maximum compressive normal stress.

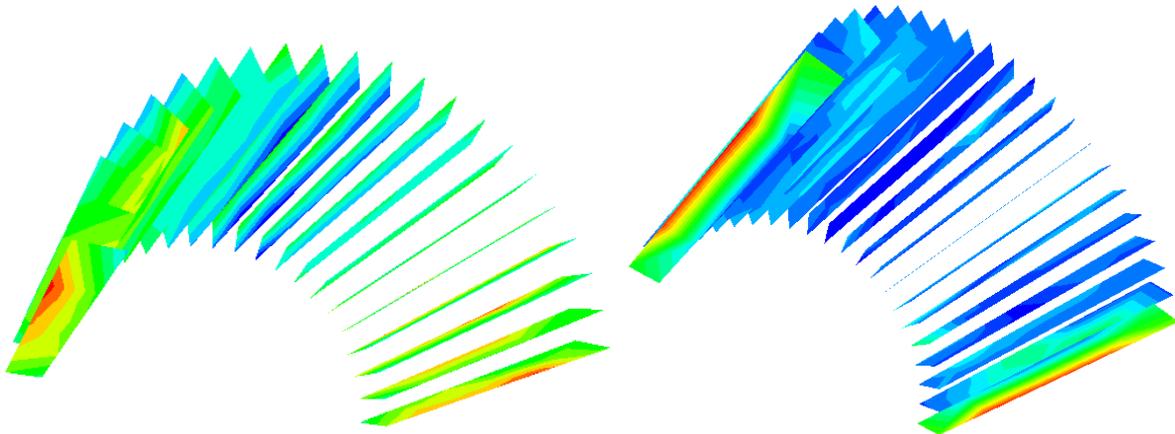


Figure 9: Normal stresses (left) and shear stresses (right) on the contacts

We can see the principal stresses on the left side of the following figure. The length of the vectors illustrate the value of the principal stress, and we can see the principal directions, too. The direction of the principal stresses are very similar to the Heymanian theory of line of thrust.

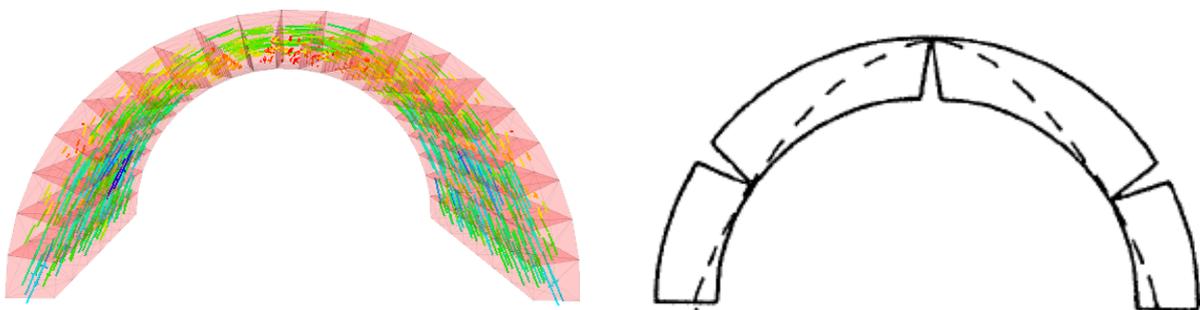


Figure 9: Principal stresses, and the so-called line of thrust

SKEW ARCH – WITH 30 DEGREE OF SKEW WITHOUT BOND PATTERN

Obviously, the skew arch doesn't need more computational time as the regular arch, because the number of the elements and the contacts don't change.

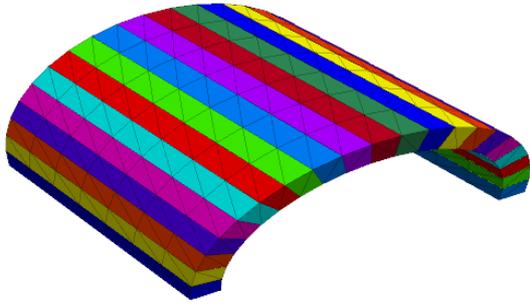


Figure 10: Geometry of skew arch

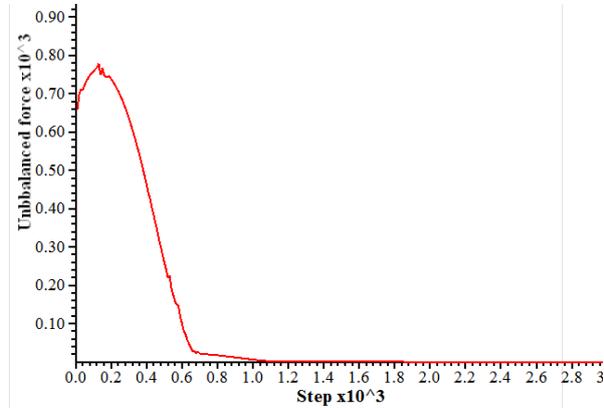


Figure 11: Unbalanced forces vs. timesteps

Firstly I checked the displacement results. The results are not totally symmetric. The maximal deflection of the crown is little bit more than the regular arch. (0,03 mm \leftrightarrow 0,035 mm)

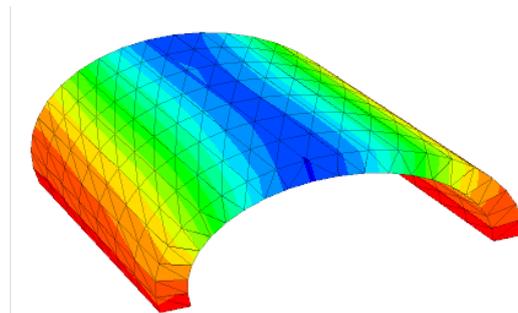


Figure 12: The deflection of the midpoint was around 0,035 mm.

Investigating the contact stresses we can see the redistribution of the internal force system.

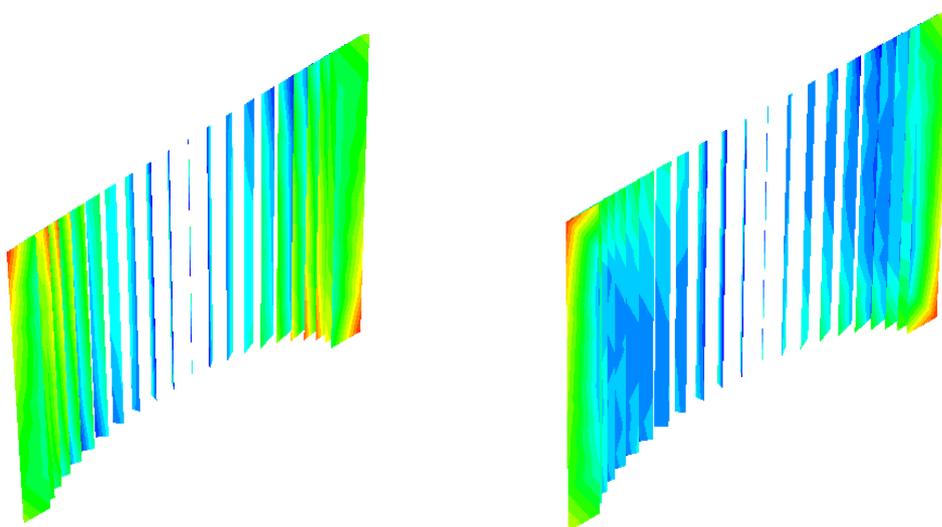


Figure 13: Contact stresses – normal and shear stresses

We can see the principal stresses on the following figure. The length of the vector illustrates the value of the principal stress, and we can see the principal directions, too. The biggest σ_3 values acting on the two opposite corner of the skew arch.

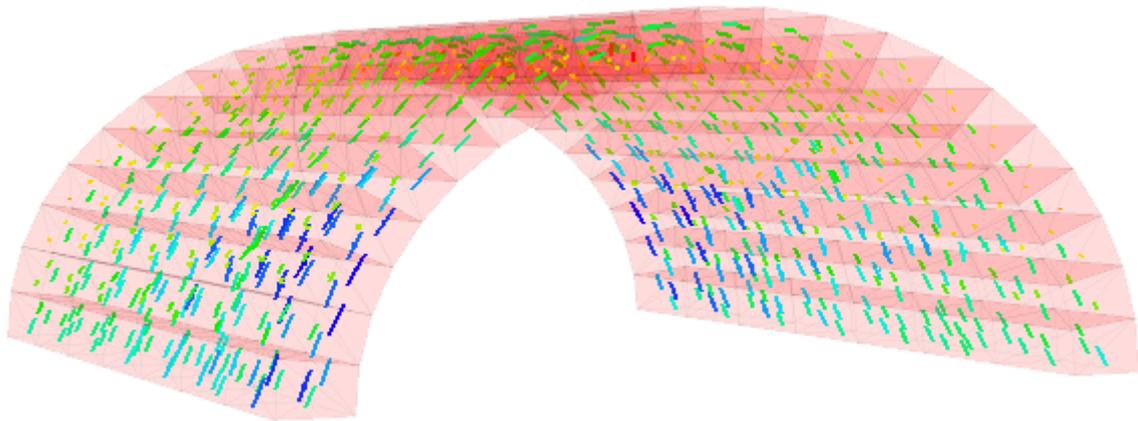


Figure 14: Principal stresses in the arch

SKEW ARCH – WITH 30 DEGREE OF SKEW AND BOND PATTERN

In this model I used the “jset” command to create blocks. The unbalanced force is not converged because there were some elements on the front and the rear face which wanted to fall out. This model need more computational time because of the increasing number of the elements and the contacts.

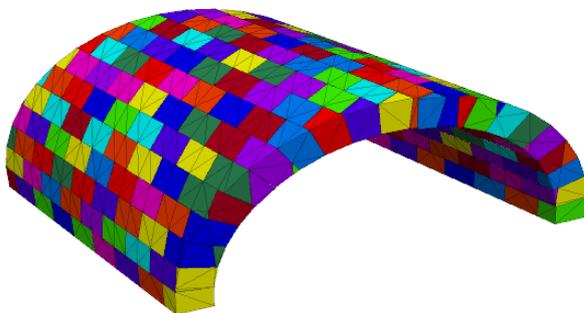


Figure 15: Geometry of skew arch

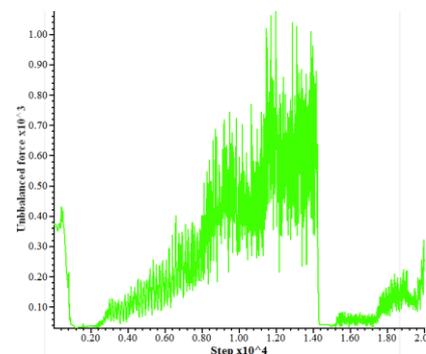


Figure 16: Unbalanced forces vs. timesteps

Firstly I checked the displacement results. The result are very similar to the previous one.

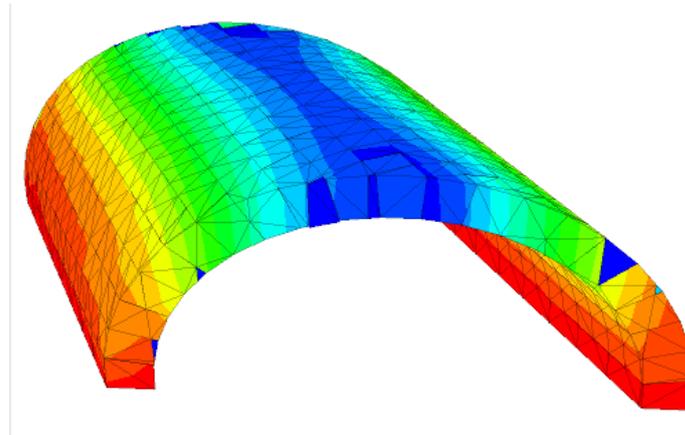


Figure 17: The deflection of the midpoint was around 0,037 mm.

Investigating the contact stresses in case of arch with bond pattern we can see the redistribution of the internal force system compare to the regular arch. If we compare the results of arches with and without bond pattern we can see almost the same stress distribution.

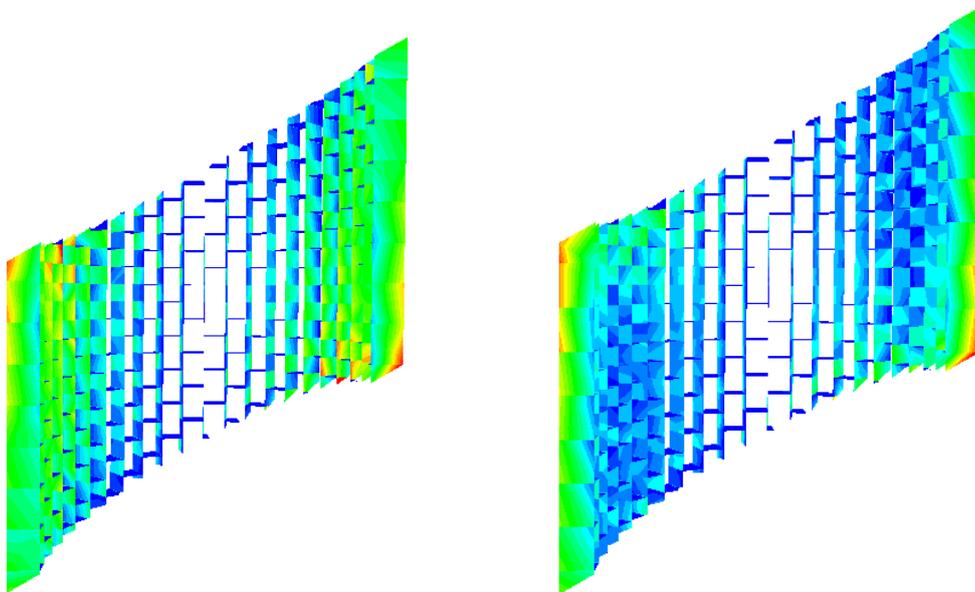


Figure 18: Contact forces – normal and shear forces

CONCLUSIONS

- I. At the moment of this homework in my opinion 3DEC isn't suitable for being the "everyday tool" of an engineer:
 - i. It doesn't have user-friendly graphical interface.
 - ii. The program doesn't help the user to import or create geometry.
 - iii. The computational time can be very long in case of bigger models.
- II. To understand the mechanical behaviour of a skew arch 3 dimensional programs are necessary because the deflection and stress results don't have symmetry around the xy middleplane.
- III. The applied arch thickness ($t \sim 0,2 * R = 40 \text{ cm}$) was enough to resist the self-weight in case of regular and skew arch too.
- IV. The redistribution of internal force system was traceable with the help of 3DEC.

REFERENCES

- [1] - Melbourne, C. & Hodgson, J.A.- 1995. - The behaviour of skewed brickwork archbridges. In Arch Bridges: Proceedings of the First International Conference of Arch Bridges: 309-320. London: Thomas Telford.
- [2] – Sarhosis, V., Oliveira, D.V., Lemos, J. Sheng, Y., Lourenco, P.B – 2013 - The Behaviour of single span stone masonry skew arches
- [3] 3DEC - 3 Dimensional Distinct Element Code - Online Manual

APPENDIX A: MAPLE CODE

Kiindulási adatok:

$r_int := 2; t := 0.4 : r_ext := r_int + t$

Angle of the arch:

$\beta := \pi;$

Width of the arch:

$w := 4;$

Angle of the skew:

$\alpha := \frac{\pi}{6} :$

Number of the elements:

$n := 21$

for i from 1 to $n + 1$ do

$koor[i][1] := evalf\left(0 - r_int \cdot \cos\left(\frac{\beta}{n} \cdot (i - 1)\right)\right);$

$koor[i][2] := evalf\left(0 + r_int \cdot \sin\left(\frac{\beta}{n} \cdot (i - 1)\right)\right);$

$koor[i][3] := evalf(\tan(\alpha) \cdot (r_ext + koor[i][1]));$

$koor2[i][1] := evalf\left(0 - r_int \cdot \cos\left(\frac{\beta}{n} \cdot (i - 1)\right)\right);$

$koor2[i][2] := evalf\left(0 + r_int \cdot \sin\left(\frac{\beta}{n} \cdot (i - 1)\right)\right);$

$koor2[i][3] := evalf(\tan(\alpha) \cdot (r_ext + koor[i][1]) + w);$

$koor[i + n + 1][1] := evalf\left(0 - r_ext \cdot \cos\left(\frac{\beta}{n} \cdot (i - 1)\right)\right);$

$koor[i + n + 1][2] := evalf\left(0 + r_ext \cdot \sin\left(\frac{\beta}{n} \cdot (i - 1)\right)\right);$

$koor[i + n + 1][3] := evalf(\tan(\alpha) \cdot (r_ext + koor[i + n + 1][1]));$

$koor2[i + n + 1][1] := evalf\left(0 - r_ext \cdot \cos\left(\frac{\beta}{n} \cdot (i - 1)\right)\right);$

$koor2[i + n + 1][2] := evalf\left(0 + r_ext \cdot \sin\left(\frac{\beta}{n} \cdot (i - 1)\right)\right);$

$koor2[i + n + 1][3] := evalf(\tan(\alpha) \cdot (r_ext + koor[i + n + 1][1]) + w);$

end do:

```

for  $i$  from 1 to  $n$  do
input[ $i$ ][1] := "polyhedron prism a";
input[ $i$ ][2] := koor[ $i$ ][1];
input[ $i$ ][3] := koor[ $i$ ][2];
input[ $i$ ][4] := koor[ $i$ ][3];
input[ $i$ ][5] := koor[ $i + 1$ ][1];
input[ $i$ ][6] := koor[ $i + 1$ ][2];
input[ $i$ ][7] := koor[ $i + 1$ ][3];
input[ $i$ ][8] := koor[ $i + 2 + n$ ][1];
input[ $i$ ][9] := koor[ $i + 2 + n$ ][2];
input[ $i$ ][10] := koor[ $i + 2 + n$ ][3];
input[ $i$ ][11] := koor[ $i + n + 1$ ][1];
input[ $i$ ][12] := koor[ $i + n + 1$ ][2];
input[ $i$ ][13] := koor[ $i + n + 1$ ][3];
input[ $i$ ][14] := "b";
input[ $i$ ][15] := koor2[ $i$ ][1];
input[ $i$ ][16] := koor2[ $i$ ][2];
input[ $i$ ][17] := koor2[ $i$ ][3];
input[ $i$ ][18] := koor2[ $i + 1$ ][1];
input[ $i$ ][19] := koor2[ $i + 1$ ][2];
input[ $i$ ][20] := koor2[ $i + 1$ ][3];
input[ $i$ ][21] := koor2[ $i + 2 + n$ ][1];
input[ $i$ ][22] := koor2[ $i + 2 + n$ ][2];
input[ $i$ ][23] := koor2[ $i + 2 + n$ ][3];
input[ $i$ ][24] := koor2[ $i + n + 1$ ][1];
input[ $i$ ][25] := koor2[ $i + n + 1$ ][2];
input[ $i$ ][26] := koor2[ $i + n + 1$ ][3];
input[ $i$ ][27] := ";";
end do:

```

$INP := Matrix(n + 2, 27, (i, j) \rightarrow input[i][j])$

With the help of the MAPLE I generated a matrix. Every row of this matrix defines one block in 3DEC.

APPENDIX B: 3DEC CODE

new

```
polyhedron prism a -2 0 0.230940108 -1.977661652 0.298084532 0.243837159
-2.373193983 0.357701439 0.015476461 -2.4 0 0 b -2 0
4.230940108 -1.977661652 0.298084532 4.243837159 -2.373193983 0.357701439
4.015476461 -2.4 0 4 ;
```

```
polyhedron prism a -1.911145612 0.589510349 0.282240213 -1.801937736 0.867767479
0.345291409 -2.162325283 1.041320974 0.137221562 -2.293374734 0.707412419
0.061560126 b -1.911145612 0.589510349 4.282240213 -1.801937736
0.867767479 4.345291409 -2.162325283 1.041320974 4.137221562 -
2.293374734 0.707412419 4.061560126 ;
```

```
polyhedron prism a -1.652477549 1.126640116 0.431582288 -1.466103743 1.360345476
0.539185256 -1.759324492 1.632414571 0.369894177 -1.982973058 1.35196814
0.240770617 b -1.652477549 1.126640116 4.431582288 -1.466103743
1.360345476 4.539185256 -1.759324492 1.632414571 4.369894177 -
1.982973058 1.35196814 4.240770617 ;
```

```
polyhedron prism a -1.246979604 1.563662965 0.665696636 -1 1.732050808
0.808290377 -1.2 2.07846097 0.692820323 -1.496375524 1.876395558
0.521707835 b -1.246979604 1.563662965 4.665696636 -1 1.732050808
4.808290377 -1.2 2.07846097 4.692820323 -1.496375524 1.876395558
4.521707835 ;
```

```
polyhedron prism a -0.730682048 1.861747498 0.963781169 -0.445041867 1.949855825
1.128695605 -0.53405024 2.33982699 1.077306596 -0.876818457 2.234096997
0.879409274 b -0.730682048 1.861747498 4.963781169 -0.445041867
1.949855825 5.128695605 -0.53405024 2.33982699 5.077306596 -
0.876818457 2.234096997 4.879409274 ;
```

```
polyhedron prism a -0.149460187 1.994407594 1.299349767 0.149460187 1.994407594
1.471931526 0.179352224 2.393289113 1.489189701 -0.179352224 2.393289113
1.282091592 b -0.149460187 1.994407594 5.299349767 0.149460187
1.994407594 5.471931526 0.179352224 2.393289113 5.489189701 -
0.179352224 2.393289113 5.282091592 ;
```

```
polyhedron prism a 0.445041867 1.949855825 1.642585688 0.730682048 1.861747498
1.807500124 0.876818457 2.234096997 1.891872019 0.53405024 2.33982699
1.693974696 b 0.445041867 1.949855825 5.642585688 0.730682048
1.861747498 5.807500124 0.876818457 2.234096997 5.891872019 0.53405024
2.33982699 5.693974696 ;
```

```
polyhedron prism a 1 1.732050808 1.962990915 1.246979604 1.563662965
2.105584657 1.496375524 1.876395558 2.249573458 1.2 2.07846097
2.07846097 b 1 1.732050808 5.962990915 1.246979604 1.563662965
```

```

6.105584657 1.496375524 1.876395558 6.249573458 1.2 2.07846097
6.07846097 ;

polyhedron prism a 1.466103743 1.360345476 2.232096038 1.652477549 1.126640116
2.339699005 1.982973058 1.35196814 2.530510675 1.759324492 1.632414571
2.401387115 b 1.466103743 1.360345476 6.232096038 1.652477549
1.126640116 6.339699005 1.982973058 1.35196814 6.530510675 1.759324492
1.632414571 6.401387115 ;

polyhedron prism a 1.801937736 0.867767479 2.425989884 1.911145612 0.589510349
2.48904108 2.293374734 0.707412419 2.709721167 2.162325283 1.041320974
2.63405973 b 1.801937736 0.867767479 6.425989884 1.911145612
0.589510349 6.48904108 2.293374734 0.707412419 6.709721167 2.162325283
1.041320974 6.63405973 ;

polyhedron prism a 1.977661652 0.298084532 2.527444134 2 0 2.540341186
2.4 0 2.771281293 2.373193983 0.357701439 2.755804832 b
1.977661652 0.298084532 6.527444134 2 0 6.540341186 2.4 0
6.771281293 2.373193983 0.357701439 6.755804832 ;

mark region 1;
jset dip 0 dd 0 spac 0.5 num 50 origin 0,0,0;
hide reg 1;

polyhedron prism a -1.977661652 0.298084532 0.243837159 -1.911145612 0.589510349
0.282240213 -2.293374734 0.707412419 0.061560126 -2.373193983 0.357701439
0.015476461 b -1.977661652 0.298084532 4.243837159 -1.911145612
0.589510349 4.282240213 -2.293374734 0.707412419 4.061560126 -
2.373193983 0.357701439 4.015476461 ;

polyhedron prism a -1.801937736 0.867767479 0.345291409 -1.652477549 1.126640116
0.431582288 -1.982973058 1.35196814 0.240770617 -2.162325283 1.041320974
0.137221562 b -1.801937736 0.867767479 4.345291409 -1.652477549
1.126640116 4.431582288 -1.982973058 1.35196814 4.240770617 -
2.162325283 1.041320974 4.137221562 ;

polyhedron prism a -1.466103743 1.360345476 0.539185256 -1.246979604 1.563662965
0.665696636 -1.496375524 1.876395558 0.521707835 -1.759324492 1.632414571
0.369894177 b -1.466103743 1.360345476 4.539185256 -1.246979604
1.563662965 4.665696636 -1.496375524 1.876395558 4.521707835 -
1.759324492 1.632414571 4.369894177 ;

polyhedron prism a -1 1.732050808 0.808290377 -0.730682048 1.861747498
0.963781169 -0.876818457 2.234096997 0.879409274 -1.2 2.07846097
0.692820323 b -1 1.732050808 4.808290377 -0.730682048 1.861747498

```

```

4.963781169 -0.876818457 2.234096997 4.879409274 -1.2 2.07846097
4.692820323 ;

polyhedron prism a -0.445041867 1.949855825 1.128695605 -0.149460187 1.994407594
1.299349767 -0.179352224 2.393289113 1.282091592 -0.53405024 2.33982699
1.077306596 b -0.445041867 1.949855825 5.128695605 -0.149460187
1.994407594 5.299349767 -0.179352224 2.393289113 5.282091592 -0.53405024
2.33982699 5.077306596 ;

polyhedron prism a 0.149460187 1.994407594 1.471931526 0.445041867 1.949855825
1.642585688 0.53405024 2.33982699 1.693974696 0.179352224 2.393289113
1.489189701 b 0.149460187 1.994407594 5.471931526 0.445041867
1.949855825 5.642585688 0.53405024 2.33982699 5.693974696 0.179352224
2.393289113 5.489189701 ;

polyhedron prism a 0.730682048 1.861747498 1.807500124 1 1.732050808
1.962990915 1.2 2.07846097 2.07846097 0.876818457 2.234096997
1.891872019 b 0.730682048 1.861747498 5.807500124 1 1.732050808
5.962990915 1.2 2.07846097 6.07846097 0.876818457 2.234096997
5.891872019 ;

polyhedron prism a 1.246979604 1.563662965 2.105584657 1.466103743 1.360345476
2.232096038 1.759324492 1.632414571 2.401387115 1.496375524 1.876395558
2.249573458 b 1.246979604 1.563662965 6.105584657 1.466103743
1.360345476 6.232096038 1.759324492 1.632414571 6.401387115 1.496375524
1.876395558 6.249573458 ;

polyhedron prism a 1.652477549 1.126640116 2.339699005 1.801937736 0.867767479
2.425989884 2.162325283 1.041320974 2.63405973 1.982973058 1.35196814
2.530510675 b 1.652477549 1.126640116 6.339699005 1.801937736
0.867767479 6.425989884 2.162325283 1.041320974 6.63405973 1.982973058
1.35196814 6.530510675 ;

polyhedron prism a 1.911145612 0.589510349 2.48904108 1.977661652 0.298084532
2.527444134 2.373193983 0.357701439 2.755804832 2.293374734 0.707412419
2.709721167 b 1.911145612 0.589510349 6.48904108 1.977661652
0.298084532 6.527444134 2.373193983 0.357701439 6.755804832 2.293374734
0.707412419 6.709721167 ;

```

mark region 2;

jset dip 0 dd 0 spac 0.50 num 50 origin 0,0,-0.25;

hide reg 2;

polyhedron prism a -8,0,-5 10,0,-5 10,0,15 -8,0,15 b -8,-2,-5 10,-2,-5 10,-2,15 -8,-2,15

```
mark region 3;
```

```
fix reg 3;
```

```
seek
```

```
hide reg 2;
```

```
prop mat=1 dens=2600 k=2.68e10 g=0.70e10
```

```
prop jmat=1 jkn 1e11 jks 1e11 jfri 40
```

```
plot block;
```

```
seek;
```

```
generate edge 0.6;
```

```
gravity 0,-9.81,0;
```

```
hist unbal id=1
```

```
cycle 1
```

```
plot hist 1 yaxis label 'Unbbalanced force'
```

```
plot contour ydisp above au
```

```
plot jointcontour sforce
```

```
plot jointcontour nforce
```

```
cycle 20000
```