Numerical Models for Structures

DEM comparison of the horizontal reaction of Romanesque and Gothic arches

Zsuzsa Borbála PAP
Supervisor: Dr. Katalin Bagi

2014.04.07.
Contents

• Introduction
• 3DEC model
  - Geometry
  - Material models
  - Boundary conditions and loading
  - Support displacements
• Results
  - Displacements
  - Contact shear force
• Conclusions
Introduction

Romanesque and Gothic arch
Aim of homework

- Software: **3DEC 5.00.164** of Itasca
- Semi-circular barrel vault of 2 m inner span, 15 cm thickness and 1 m length,
- Pointed vault having the same inner span, length and selfweight as the semi-circular vault

First task – find the equilibrium under selfweight:
- Both vaults \( H \) horizontal component of reactions:

Second task – move the supports:
- Several support displacement \( (e) \) measure \( H \)
- Compare the \( H(e) \) diagrams of the two structures
**Romanesque arch**
- Make them larger → fell down
- Horizontal reaction push the wall → collapse
- Small windows → dark and dreary
- Timber roofs → burn down

**Gothic arch**
- The centerline of pointed arches follows more closely the compression forces → stronger
- Weight → flying buttresses
- Large windows → filled with light
- Can have bigger span
3DEC model
Geometry

- **Semi-circular arch:**
  - 2 m inner span, 15 cm thickness, 1 m length
  - 17 blocks
  - support blocks: 30cmx40cmx1m

- **Pointed vault:**
  - same span, length, selfweight → 11.75 cm
  - 17 blocks
  - support blocks: 30cmx40 cmx1 m
GEOMETRY

17+2 blocks of the Romanesque and Gothic arch
Material models

- **Block:**
  - rigid
  - 2600 kg/m\(^3\) (limestone, sandstone, marble)

- **Contacts:**
  - Coulomb frictional
  - Shear stiffness (jks) and normal stiffness (jkn) are \(10^{12}\) N/m\(^2\)
  - Friction angle: 50°
Boundary conditions loading

- Support blocks: fixed
- Gravity in \(-y\) vertical direction

Support displacements

- Velocity
- Length of time step
- Number of time steps
Results
Displacements

- Romanesque
Displacements

- Gothic
# Displacements

<table>
<thead>
<tr>
<th></th>
<th>Romanesque arch</th>
<th>Gothic arch</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum vertical</td>
<td>-7.1781*10^{-7}</td>
<td>-7.9286*10^{-7}</td>
<td>9.47 %</td>
</tr>
<tr>
<td>displacement [m]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum horizontal</td>
<td>-2.6427*10^{-7}</td>
<td>-6.0149*10^{-7}</td>
<td>56 %</td>
</tr>
<tr>
<td>displacement [m]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Gothic > Romanesque
- Very small displacements!
Horizontal reaction

- Plot – problems
- Solution: check the normal forces

- Contact shear force (N)

<table>
<thead>
<tr>
<th>Romanesque</th>
<th>Gothic</th>
</tr>
</thead>
<tbody>
<tr>
<td>2385.75</td>
<td>1628.39</td>
</tr>
</tbody>
</table>
H-e diagram

- Romanesque
H-e diagrams

- Gothic
H-e diagrams

- Comparison

<table>
<thead>
<tr>
<th>Force (N)</th>
<th>Gothic arch</th>
<th>Romanesque arch</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>0.00125</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>0.0025</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>0.00375</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>0.005</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>0.0075</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>0.01</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>0.015</td>
<td>40%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Displacement (mm)
Conclusions
Conclusions

- Horizontal reaction of the Gothic arch is much smaller than the reaction of the Romanesque arch.
- If the arches are under selfweight the horizontal reaction of the Gothic arch is 31% smaller.
- When we are on the constant section of the force-displacement diagram the difference become even larger (40%).
- Structure of the Gothic arch has an advantageous effect on the support structure, because the horizontal force component is smaller.
- We have fewer problems with the stability of the supporting walls, so actually we can have thinner walls with bigger windows.
THANK YOU FOR YOUR ATTENTION!