Free vibration of piecewise linear continuous systems with a regular state

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Introduction

In this study the effect of a multi-freedom constraint on the free vibration of a piecewise linear systems is analysed. The examined structures are the members of a family of simply supported beams that are also strengthened by a block-and-tackle suspension system [1]. Piecewise linear stiffness occurs often in the engineering practice: e.g. an opening-closing gap, a spiral spring in compression, or a tightening-slackening cable can be modeled this way, where the change in the stiffness depends on the displacements.

The analysed model allows to consider the cable as a one-parameter force system. By separating the behaviour into active and passive state of the cable, linear modal analysis becomes applicable. The change between the active and passive states can be handled by a transformation matrix, that represents the connection between the active and passive set of normal modes.

The aim of this project is to determine the limits of existence of periodic vibration modes for such structures.

Model

Geometry:

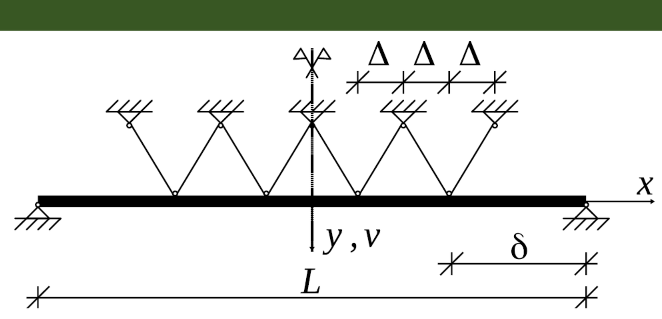


Figure 1. Geometry and parameters of the structure

Parameters:

- $2c$: number of pulleys
- μ : specific mass
- EI : stiffness of the beam
- u : displacement function
- λ : frequency parameter

Assumptions:

- Small displacements
- Euler-Bernoulli beam theory
- Prismatic beam
- Constant bending stiffness, mass
- Constant cable force
- Inextensible cable
- Massless, small pulleys
- Placed equidistantly, symmetrically

PDE of free vibration: $\mu \ddot{u}(x,t) + EIu''''(x,t) = 0$

Modal time-history analysis

Initial active mode:

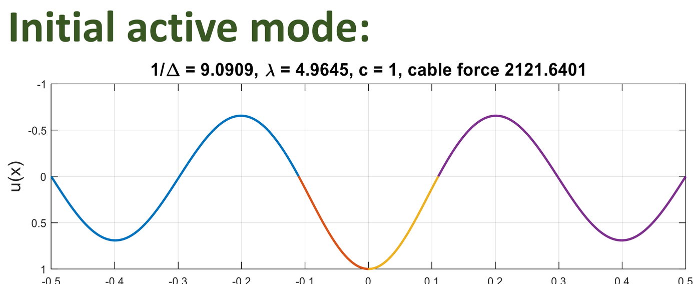


Figure 2.A) Inital active mode: c = 1, Δ = 0.11, n = 2

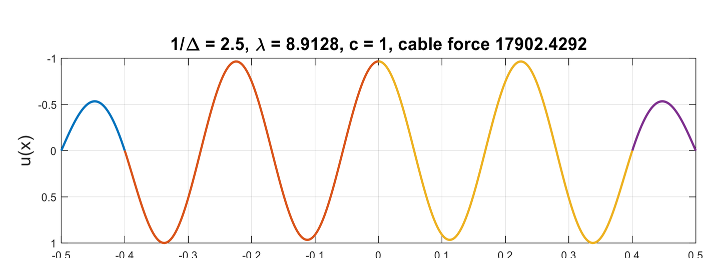


Figure 2.B) Initial active mode: c = 1, Δ = 0.4, n = 2

Starting from an active normal mode [2], the motion of the beam can be followed through several state changes. When the beam reaches the $u(x,t) = 0$ equilibrium state, the cable enters passive state. Since each passive normal modes consist of sine half-waves, in a regular case the passive modal velocities are exactly the opposite at the beginning and at the end of the state. This makes it possible to return into the initial active mode after a half period of passive state. This process requires that the cable remains slacked during the whole passive period, e.g. no early change occurs.

In the A) case this condition is fulfilled, thus this motion is periodic (a Regular Nonlinear Normal Mode). In the B) case the system cannot run through the passive half period, the cable is tightened earlier, causing a non-periodic motion.

Displacements:

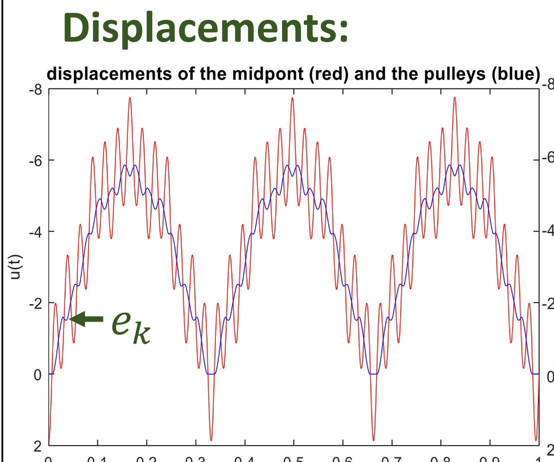
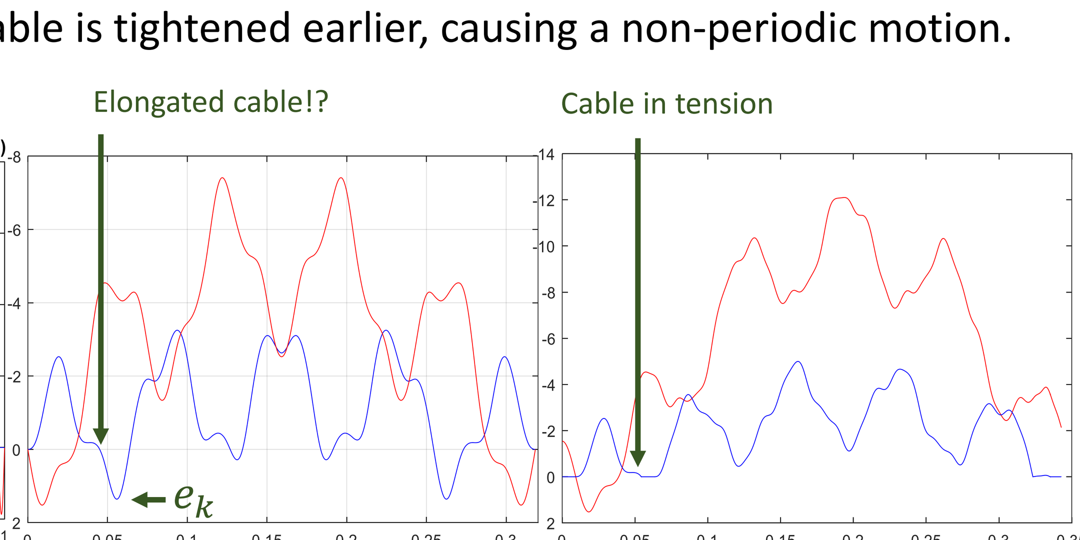


Figure 3.a) motion starting from the 2.A) mode

A) Periodic mode



3.b) motion starting from the 2.B) mode without... and 3. c) with considering the early change

B) Non-periodic mode

Phase space

The motion can also be depicted in the phase space. The figure shows the displacement and the velocity of the middle point of the beam. In the periodic case after fourteen state changes the motion is still clearly periodic (Figure 4.A contains overlapping curves). For the nonperiodic case, the motion is more complicated (Figure 4.B).

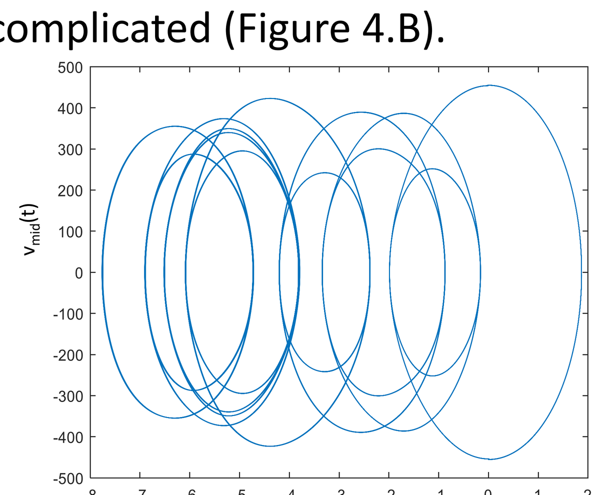
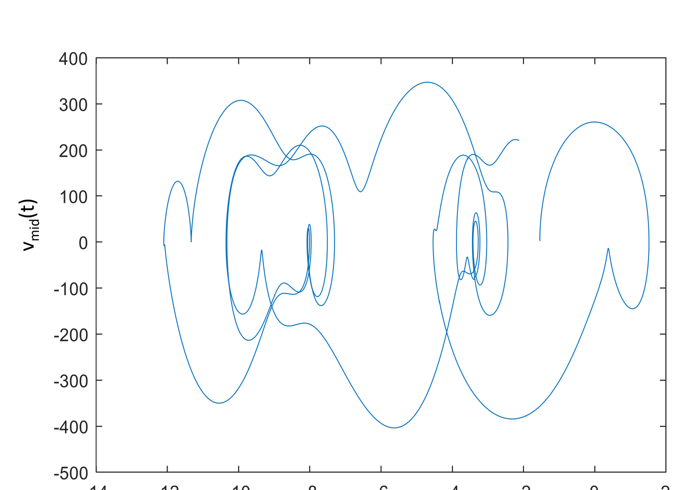


Figure 4.A) motion of the midpoint in the phase space in the periodic case, and



4. B) in the nonperiodic case

Frequency map

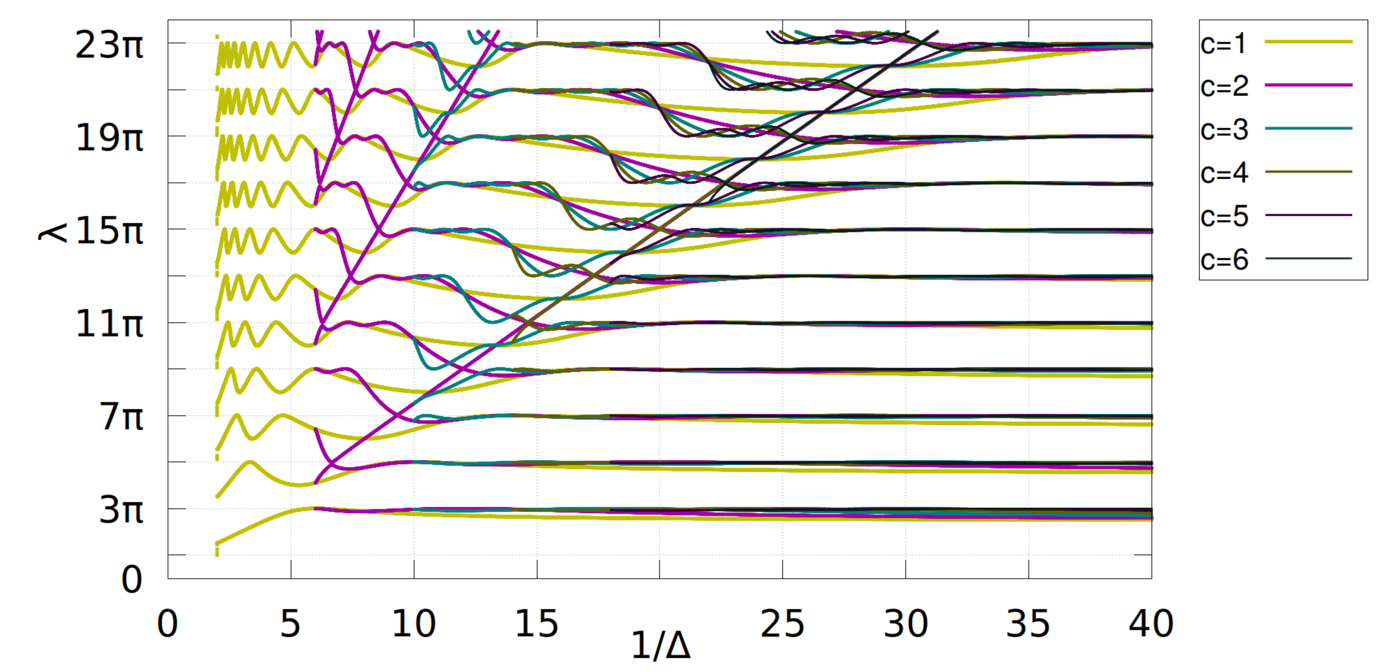


Figure 5. Frequency map: frequency parameters for a wide set of the suspended beams

To analyse the whole family of such structures and not only a few examples, the frequency map (Figure 5.) was created [2], that shows the λ frequency parameter of the active mode shapes for each member described by Δ and c parameters. The natural circular frequency of any member of the family can be caulated using this frequncy map and the following formula,

$$\omega_{0,i} = \frac{\lambda_i^2}{L^2}\sqrt{\frac{EI}{\mu}}$$

where: λ_i - is the frequency parameter of the i-th mode
L – is the legth of the beam
EI – is the bending stiffness of the beam
μ – is the specific mass of the beam

Stationary points

To determine if the motion of a certain member of the family is periodic or not, the concept of stationary points was introduced [3]. The function $e(t)$ describes the elongation of the cable. From this function e^k is defined as the value of this function at the first t_k time instant, where $\dot{e}(t_k) = 0$, and $e(t_k)$ is the maximum. Thus, to avoid the early change of state, this e^k value needs to be negative.

The figures below (Figure 6.) show the e^k maximum stationary values in view of the Δ and c parameters, for several initial active modes (n). For negative e^k values the motion is regular and therefore periodic, otherwise the motion is irregular.

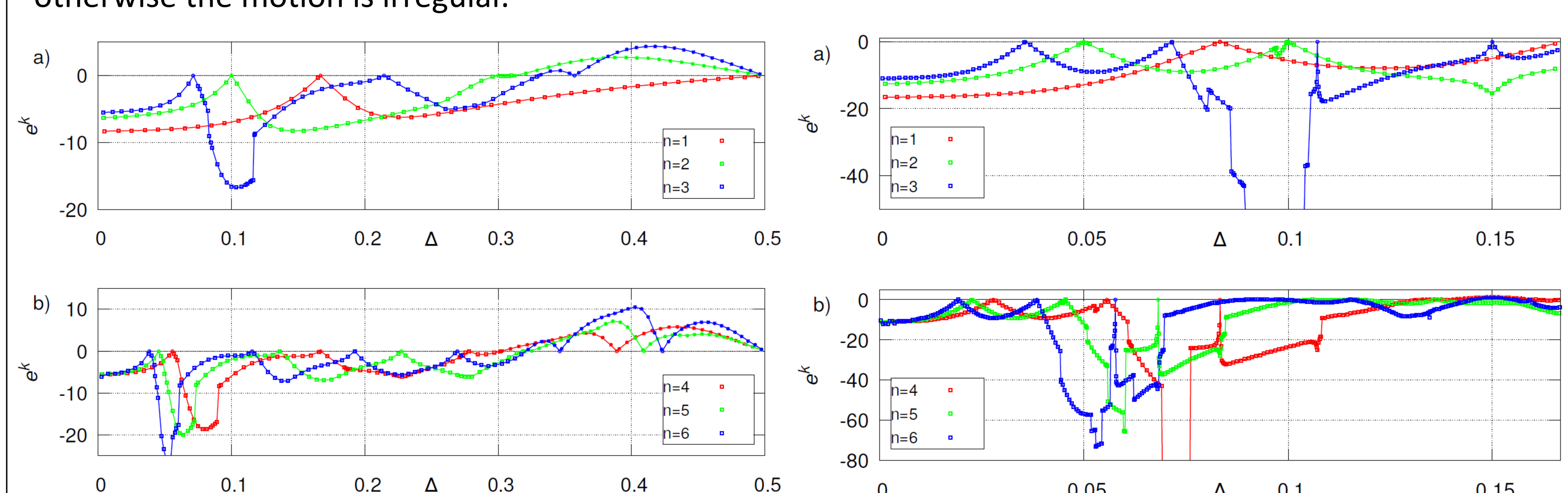


Figure 6.a) stationary points for different Δ parameters for c = 1, 6.b) stationary points for c = 2 and

Conclusions

Discussion:

- The graphs show that independent of the position of the suspension points, the first mode is always periodic. Futhermore, for these cases the first $2c-1$ modes are always periodic.
- Even for the higher modes typically the smaller Δ values result in periodic motion, and the higher Δ values induce non-periodic behaviour. The observation here is that if the last pulley is in the $0.0-0.3$ distance from the centre of the beam, then the e^k values remain non-positive.
- The jumps in the functions are due to the change of the position of the e^k values, hence the stationary point of the $e(t)$ function occurs at a different t_k time instant.

Further research:

- Analysis of the forced vibration of the structure.
- Different positions of the pulleys: for exmple when there are a very high number of pulleys placed on a distinct part of the beam, they can modeled as a distributed support.
- To follow the non-regular but periodic cases different parameters should be used: e.g. how much time does the structure spend in active or in passive state?

References

[1] A. Kocsis, R. K. Németh, B. Turmunkh. Dynamic analysis of a beam on block-and-tackle suspension system: A continuum approach. *Engineering structures*, (2015)

[2] B. Geleji, R. K. Németh. Függesztett gerendacsalád szabadrezgése I.-II. (in Hungarian). *Építés-Építészettudomány*. (2018)

[3] R. K. Németh, B. B. Geleji. Periodic nonlinear normal modes of piecewise linear continuous systems with a regular state. submitted to *Periodica Polytechinca Civil Engineering*.

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